I

A) $f(x) = ax^2 + bx + c$

$c = 2$

$f(1) = a + b + 2 \overset{\text{set}}{=} 0$

$f(2) = 4a + 2b + 2 \overset{\text{set}}{=} 1$

$4a + 2(-a-2) = -1$

$4a - 2a - 4 = 3$

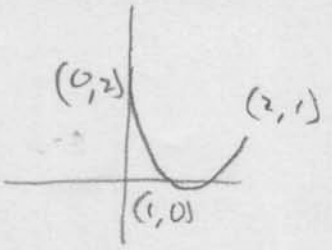
$2a = 3$

$a = \frac{3}{2}$

$\frac{3}{2} + b + \frac{4}{2} = 0$

$b = -\frac{7}{2}$

$f(x) = \frac{3}{2}x^2 - \frac{7}{2}x + 2$



b) $\frac{1-1}{0-2} = m = -\frac{1}{2}$

$\therefore y = -\frac{1}{2}x + 2$

c) "difference in y-values."

$d(x) = -\frac{1}{2}x + 2 - \left(\frac{3}{2}x^2 - \frac{7}{2}x + 2\right)$

$= -\frac{1}{2}x + 2 - \frac{3}{2}x^2 + \frac{7}{2}x - 2$

$d(x) = 3x - \frac{3}{2}x^2$

$d'(x) = 3 - 3x \overset{\text{set}}{=} 0$

$x = 1$

$d''(x) = -3$

$\therefore x = 1$ max f" test.

$f(1) = 0$)

Point is $(1, 0)$ where vertical distance to l is greatest.

II

$p = \frac{30x^2}{(1+x^2)^2} \quad x \geq 0$

$p'(x) = \frac{(1+x^2)^2 60x - 30x^2 \cdot 2(1+x^2)(2x)}{(1+x^2)^4}$

$\frac{60x(1+x^2)}{(1+x^2)^4}\left(1+x^2-2x^2\right)$

$\frac{60x}{(1+x^2)^3}(1-x^2) \overset{\text{set}}{=} 0$

$x = 1$

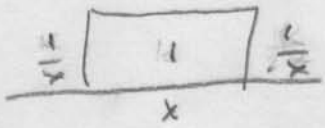
p': 0 $-$ 0 $+$ 0 $-$ (at -1, 0, 1)

max f' test

Since $x = 0, -1$ are not in obvious domain.

most people will be infected 1 month after the start and $\frac{30}{4} = \frac{15}{2} = 7\frac{1}{2}$ Percent will be infected.

III



$C(x) = 3x + x + 2 \cdot \frac{1}{x}$

$C(x) = 4x + \frac{2}{x}$

$C'(x) = 4 - \frac{2}{x^2} \overset{\text{set}}{=} 0$

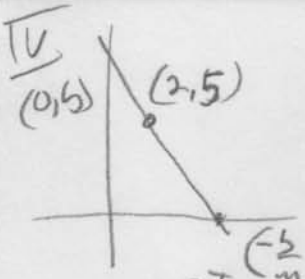
$x^2 = \frac{1}{2}$

$x = \frac{1}{\sqrt{2}}$

$C'' = \frac{4}{x^3} > 0 \therefore$ min f" test.

min cost is when the "Highway side" is $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ of a Km long. width will be $\sqrt{2}$ of a Km.

IV



$y = mx + b$

$5 = 2m + b$

$b = 5 - 2m$

$y = mx + b \overset{\text{set}}{=} 0$

$x = -\frac{b}{m}$

$A = \frac{1}{2} b h$

$A = \frac{1}{2}\left(-\frac{b}{m}\right)(b)$

$= \frac{-b^2}{2m}$

$A(m) = \frac{-(5-2m)^2}{2m}$

$= \frac{-25 + 4m - 4m^2}{2m}$

$= -\frac{25}{2m} + 2 - 2m$

$A'(m) = \frac{25}{2m^2} - 2 \overset{\text{set}}{=} 0$

$4m^2 = 25$

$m = \pm\frac{5}{2}$ 5 pts

$A''(m) = -\frac{25}{m^3} > 0$

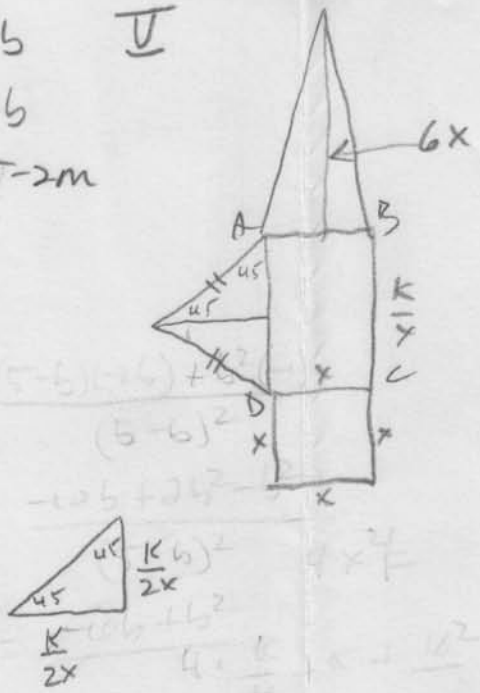
for $m < 0$ ∴,

$m = -\frac{5}{2}$ f "cst,

max area when slope is $-\frac{5}{2}$

$3 - 1\frac{2}{3}$

V



Area of ABCD = K.

$A(x) = \frac{1}{2} x \cdot 6x + K + x^2 + \frac{1}{2} \cdot \frac{K}{x} \cdot \frac{K}{2x}$

Tall Δ Rect sq RT Δ.

$A(x) = 3x^2 + K + x^2 + \frac{K^2}{4x^2}$

$= 4x^2 + K + \frac{K^2}{4x^2}$

$A'(x) = 8x - \frac{K^2}{2x^3} \overset{\text{set}}{=} 0$

$16x^4 = K^2$

$x^4 = \frac{K^2}{16}$

$x = \frac{\sqrt{K}}{2}$

$A''(x) = 8 + \frac{3K^2}{2x^4} > 0 \; \forall x > 0$

∴, min at $x = \frac{\sqrt{K}}{2}$

$4x^2 = 4 \cdot \frac{K}{4} = K$

$x = \frac{\sqrt{K}}{2}$

∴, $A\left(\frac{\sqrt{K}}{2}\right) = K + K + \frac{K^2}{K}$

$= 3K.$

The minimum area enclosed is 3K when $x = \frac{\sqrt{K}}{2}$.

Extra.

There was a total of 5 sandwiches for 3 people so each got $1\frac{2}{3}$ sandwiches.

Jake gave up $\frac{1}{3}$ of a sandwich.

Jasper gave up $1\frac{1}{3} = \frac{4}{3}$ sandwiches.

$\frac{5}{3}$ sandwiches = \$5 ∴, each $\frac{1}{3}$ = \$1

so Jake got \$1.

and Jasper got \$4.00

12/7/2000

100	0
90	4
80	3
70	8
60	4
59	7

94

1721

24

20

71.7

Advanced Placement Calculus

Name Col Ch 6 Ap p

Per _____ Date 2000

Copy original problem.

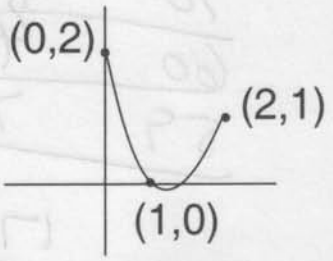
Convince *me* that **you** understand the concept*!*

No Calculators! Each section is worth 20 points.

Hint: Draw properly labeled, useful figures. Be very clear in your reasoning and answer with a concise, meaningful sentence which indicates that you understand all the concepts involved. Finally, don't waste time justifying mathematics which are incomplete and rambling. Do the job requested and move on.

Chapter 6 Applications Exam

I A cable of a suspension bridge hangs in the shape of a parabola (which is the graph of a second-degree polynomial). Consider a cable that joins two points at (0, 2) and (2, 1), and passes through the point (1, 0).



A) Let $f(x) = ax^2 + bx + c$ be the "second-degree polynomial". Obviously, $f(0) = 2$ means $c = 2$. Solve for the values of a and b.

B) Let ℓ be the line joining (0, 2) and (2, 1). Write the equation of ℓ .

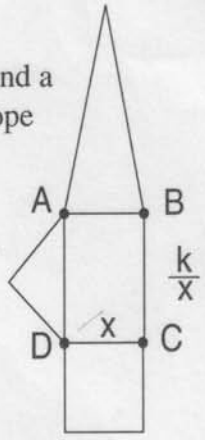
C) Find the point on the cable whose vertical distance from ℓ is the greatest.

II In a certain community a certain epidemic spreads in such a way that x months after the start of the epidemic, p percent of the population is infected where, $p = \dfrac{30x^2}{(1+x^2)^2}$. In how many months will the most people be infected and what percent of the population is this?

III A horse breeder plans to set aside a rectangular region of one square kilometer for horses and wishes to build a wooden fence to enclose the region. since one side will run along a well-traveled highway, the breeder decides to make the side more attractive, using wood that costs 3 times as much per meter as the wood for the other sides. What dimensions will minimize the cost of the fence?

IV A right triangle is formed in the first quadrant by the positive x-axis, the positive y-axis and a line which passes through the point (2, 5). Of all possible right triangles, what is the slope of the line which makes up the hypotenuse of the triangle which contains the most area?

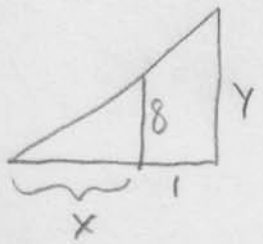
V The figure consists of a rectangle labeled $ABCD$ which has a fixed area k, an isosceles triangle with base AB and height six times the base, the square with side DC and the isosceles right triangle with hypotenuse AD. Determine the maximum or minimum possible area of the entire figure relative to the area of the rectangle. Identify whether the area found is a maximum or minimum. (Justify, of course.)



Extra Credit ---------- 5 pts ----------

Jake, Jasper, and Jed went on a picnic. Jake brought two sandwiches and Jasper brought 3 sandwiches, but Jed forgot to bring any sandwiches. The three shared the food equally and Jed paid Jake and Jasper a total of five dollars for the sandwiches that they all shared. How much money did Jake get? How much money did Jasper get? Explain.

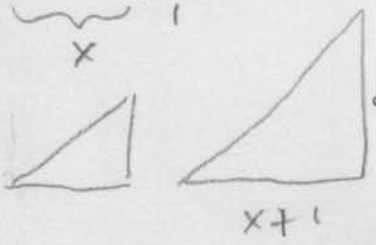
I



$\frac{x}{8} = \frac{x+1}{y}$

$y = 8\frac{(x+1)}{x}$

$y = 8 + \frac{8}{x}$



$d = \sqrt{(x+1)^2 + 64\left(1+\frac{1}{x}\right)^2}$

$d'(x) = \frac{2(x+1) + 64 \cdot 2\left(1+\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)}{2\sqrt{\quad}}$

$(x+1) - 64\left(\frac{x+1}{x^3}\right)$

$\frac{(x+1)(x^3-64)}{x^3\sqrt{\quad}} \overset{\text{set}}{=} 0$

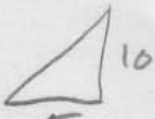
min

f' cost

$- \; 0 \; +$

$0 \qquad 4$

$x = 4$



$d = \sqrt{25+100} = \sqrt{125}$

$d = 5\sqrt{5}$

The shortest ladder would be $5\sqrt{5}$ long.

II

$P = \frac{E^2 R}{(r+R)^2}$

$P' = \frac{(r+R)^2 E^2 - E^2 R \cdot 2(r+R)}{(r+R)^4}$

$\frac{E^2(r+R)}{(r+R)^4}\left[r+R-2R\right]$

$\frac{E^2}{(r+R)^3}\left[r-R\right] \overset{\text{set}}{=} 0$

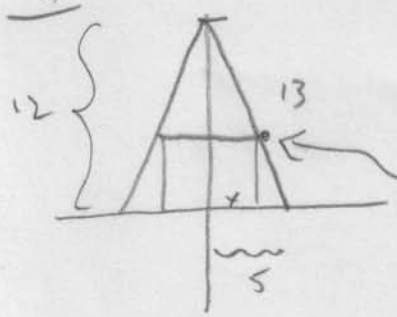
$r = R.$

$+ \; 0 \; -$

$0 \qquad R$

max power is consumed when the external resistance R is equal to the internal resistance r.

III



$y = -\frac{12}{5}x + 12$

$A = 2x \cdot \left(\frac{-12}{5}x + 12\right)$

$= \frac{-24x^2 + 24x}{5}$

$-\frac{48}{5}x + 24 \overset{\text{set}}{=} 0$

$x = 24 \cdot \frac{5}{48}$

$= \frac{5}{2}$

$A'' = -\frac{48}{5} < 0$

max f'' cost.

$16?$

-24

$+24$

$(x \quad -\frac{12}{5}x + 12)$

The rectangle is

$2 \cdot \frac{5}{2} \times$

$5 \times 6.$

4

$-\frac{12}{5} \cdot \frac{5}{2} + 12$

6

IV $f(x) = \tan x$ $\quad \pi^c = 180^\circ$ $\quad \tan 46 \approx \tan\frac{\pi}{4} + \sec^2\frac{\pi}{4} \cdot \frac{\pi}{180}$

$f'(x) = \sec^2 x$ $\quad \frac{\pi}{180} = 1^\circ$ $\quad 1 + 2 \cdot \frac{\pi}{180}$

$x = \frac{\pi}{4}$ $\quad 46^\circ = 45^\circ + 1^\circ$ $\quad 1 + \frac{\pi}{90}$

$\Delta x = \frac{\pi}{180}$ $\quad \frac{\pi}{4} + \frac{\pi}{180}$ $\quad$ So $\tan 46^\circ \approx 1 + \frac{\pi}{90}$.

V

$A = \frac{1}{2}(12-x)\sqrt{x^2 - (12-x^2)}$

$= (12-x)\sqrt{x^2 - 144 + 24x - x^2}$

$= (12-x)\sqrt{24x - 144}$

$A' = -\sqrt{24x-144} + \frac{(12-x)\cdot 12}{\sqrt{24x-144}}$

$= \frac{-(24x-144) + 144 - 12x}{\sqrt{24x-144}}$

$= \frac{-24x + 144 + 144 - 12x}{\sqrt{\quad}}$

$-36x + 288 = 0$ $\quad x = 8$

$+ \quad 0 \quad -$

$0 \qquad 8$

max f' at $x = 8$.

Area.

The triangle turns out to be $8 \times 8 \times 8$ is equilateral.

12/13/2002

Dana Williams 93
Lisa Irwin 92
Travis Tackett 89
Julie Wesp 82
Deanna Strader 82
Mike Tuck 69
Tom Valencia 67
Lauren Hylton 61
Jeff Mohr 0 } didn't
Bryan Burnley 0 } turn in.

add the comment on EC

good exam.
graded carefully. most showed concept understanding but had arithmetic errors.

Advanced Placement Calculus AB

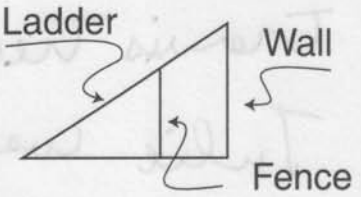
Name Cal Ch6 Ap

Per _____ Date 2000

Convince *me* that **you** understand the concept!
No Calculators, of course.
Each section is worth 20 points.

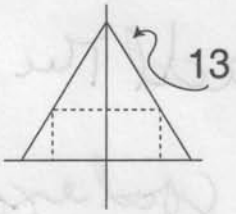
Chapter 6 Applications Make Up Exam

I A ladder is to reach over an 8 foot high fence and touch a wall which is 1 foot beyond the fence. What is the length of the shortest ladder that can be used?



II A direct current generator has an electromotive force of E volts and an internal resistance of r ohms, where E and r are constants. If R ohms is the external resistance, the total resistance is $(r+R)$ ohms and if P watts is the power (in terms of the variable R) then $P=\dfrac{E^2R}{(r+R)^2}$. Show that the most power is consumed when the external resistance is equal to the internal resistance.

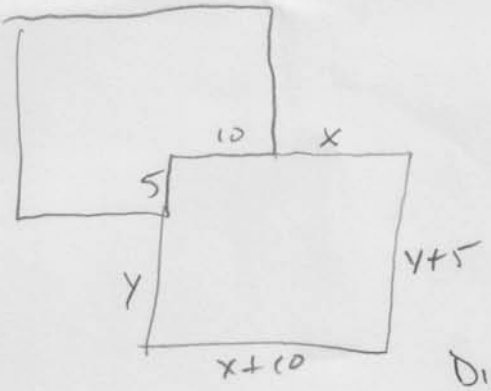
III Given the isosceles triangle with sides 13, 13, and 10. What are the dimensions of the rectangle with largest area which can be inscribed in the triangle?



IV Using differentials, compute the approximation for $\tan 46°$. Do not use decimals. (Hint: $\Delta x \neq 1$)
think RADIANS.

V Find the sides of the isosceles triangle of maximum area if the perimeter is 24.

I



$2x+2y+15=85$

$2x+2y=70$

$x+y=35$

$y=35-x$

$A=L\,W$

$A=(x+10)(y+5)$

$A(x)=(\overbrace{x+10}^{10+x})(40-x)$

$=400+30x-x^2$

$A'=30-2x \overset{\text{Set}}{=} 0$

$x=15.$

$y=20.$

$A''=-2<0 \therefore$ max f'' test.

Dimensions should be 25×25

II

$S=-K\overset{u}{x^2}\overset{v}{\ln x} \quad 0<x<1$

$S'=-2Kx\ln x-Kx^2\frac{1}{x}$

$=-2Kx\ln x-Kx$

$=-Kx(2\ln x+1)\overset{\text{Set}}{=}0$

$x=0 \quad 2\ln x=-1$

$\ln x=\frac{-1}{2}$

$x=e^{-\frac{1}{2}}=.6065$

$S''=(2\ln x+1)(-K)-Kx\cdot\frac{2}{x}$

$-K(2\ln x+1+2)$

$-K(2\ln x+3)$

$S''(e^{-\frac{1}{2}})=-K(2\cdot(-\frac{1}{2})+3)$

$-K(2)$

$-2K<0 \therefore$ max f'' test.

when the ratio of the core of the cable to the thickness of the covering is $\frac{1}{\sqrt{e}}$ the signal speed is fastest.

III

a) $600-12(x-24)$

b) $A(x)=x(600-12(x-24))$

$=x(600-12x+288)$

$=x(888-12x)$

$=888x-12x^2$

c) $A'(x)=888-24x \overset{\text{Set}}{=} 0$

$x=37$

$24\overline{)888}$ (37)

72

168

168

$A''=-24<0 \therefore$ max at $x=37$

f'' test.

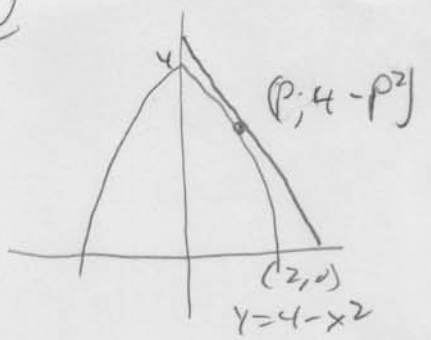
He should plant 37 trees per acre.

which would produce 16,428 apples

originally (24 trees/acre) 14,400 apples

overall increase in production of 2028 apples per acre.

IV



Need axis intercepts of tangent line. No line -15

$y' = -2x = m$ | $x = p$ → $-2(p)$ y intercept p^2+4

only have $-2x$ m | $-5x$ intercept $\frac{p^2+4}{2p}$

$y - (4-p^2] = -2p(x-p)$ pts.

$y - 4 + p^2 = -2px + 2p^2$

$y = -2px + p^2 + 4$

set $0 = -2px + p^2 + 4$

$2px = p^2 + 4$

$x = \frac{p}{2} + \frac{2}{p}$

$= \frac{p^2+4}{2p}$

$A(p) = \frac{1}{2}(p^2+4)\left(\frac{p^2+4}{2p}\right)$

$A(p) = \frac{(p^2+4)^2}{4p}$ (u, v)

$A'(p) = \frac{4p \cdot 2(p^2+4) \cdot 2p - (p^2+4)^2 \cdot 4}{16p^2}$

$= \frac{4(p^2+4)[4p^2 - p^2 - 4]}{16p^2}$

$= \frac{(p^2+4)[3p^2-4]}{4p^2}$ set $= 0$

$3p^2 - 4 = 0$

$p^2 = \frac{4}{3}$

$p = \frac{2}{\sqrt{3}}$ (Reject Neg. $p > 0$.)

	−	0	+
		$\frac{2}{\sqrt{3}}$	

0 2

minimum at $\frac{2}{\sqrt{3}}$ f test.

$A\left(\frac{2}{\sqrt{3}}\right) = \frac{\left(\frac{4}{3}+4\right)^2}{\frac{8}{\sqrt{3}}}$

$\frac{16 \cdot 16}{3 \cdot 3} \cdot \frac{\sqrt{3}}{8}$

$\frac{32\sqrt{3}}{9}$

6.158

EXTRA.

$R(x) = \overset{u}{k}x\overset{v}{(m-x)}$

$R'(x) = (m-x)k - kx \overset{\text{set}}{=} 0$

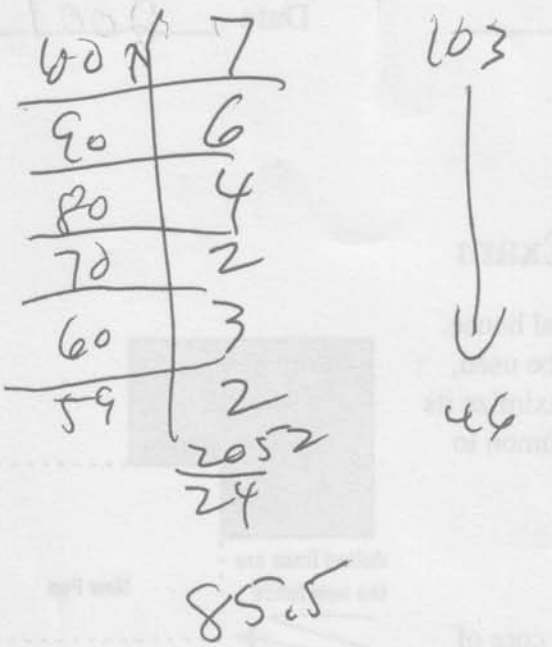
$m - x - x = 0$

$2x = m$

$x = \frac{1}{2}m$

$R''(x) = -2k < 0$ ∴ max f" test.

The spread is its highest when $\frac{1}{2}$ of the population is infected (as you would expect).



Students Did
Very well!
This wasn't a real difficult
but to do well, they
had to be focused.
No changes necessary.

Advanced Placement Calculus

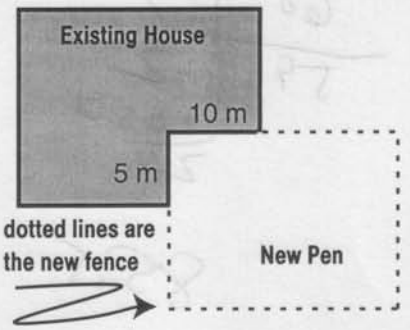
Name Calc Ch 6 App

Per _____ Date 2001

Copy original problem.
Convince *me* that **you** understand the concept*!*
Each section 25 points. All extrema problems require proper justification. If decimals are used, they must be accurate to 3 places right of the point.

Chapter 6 Applications Exam

I A rectangular outdoor pen is to be added to an existing animal house. See the figure on the right. If 85 meters of new fence are to be used, what should be the dimensions of the new pen in order to maximize its area. Of course, no fence will be needed along the walls common to the new pen and the old animal house.



II In a transatlantic cable, let x be the ratio of the radius of the core of the cable to the thickness of the covering for the cable. The speed s of a signal passing through the cable is given by: $s = -kx^2 \ln x$ for $0 < x < 1$ where k is a positive constant. What will maximize s ?

III The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will yield 600 apples per year. For each additional tree planted per acre, the number of apples produced by each tree decreases by 12 apples per year due to competition for water, nutrients, sunlight, etc. (Thus, for example, if 25 trees were planted per acre, then *each* tree would produce $600 - 12(25 - 24) = 588$ apples per year.)

a) If x is the number of trees per acre, write an expression for the number of apples which will produced by each tree (i.e.. apples per tree).

b) Continuing from (a), write a function for the number of apples *per acre*. (Remember, there are x trees per acre.)

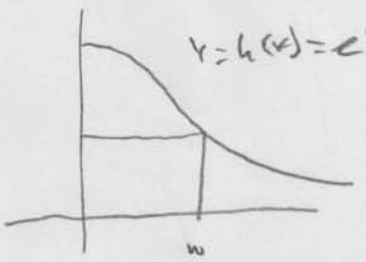
c) Determine the number of trees per acre which will produce the most apples per acre. How many more apples will be produced at this level?

IV At what first-quadrant point on the parabola $y = 4 - x^2$ does the tangent, together with the coordinate axes, determine p so the triangle has minimum area (no decimals)? What is that area (decimals allowed here)? (Hint: let the tangent point have coordinates $(p, 4 - p^2)$)

Extra Credit ---------- 5 pts ----------

In one model of the spread of a contagious disease among members of a population of M people, the incidence of the disease, R, measured as the number of new cases per day is given in terms of the number, x, of individuals already infected by: $R(x) = kx(M - x)$ where k is some positive constant. How many individuals in the population are infected when the incidence, R is the highest?

I

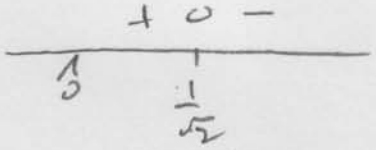


max rectangle: $A(w) = w(e^{-w^2})$

$A'(w) = e^{-w^2} + w(-2w)e^{-w^2}$

$= e^{-w^2}(1-2w^2) \overset{Set}{=} 0$

$w = \frac{1}{\sqrt{2}}$



max ~~test~~.

PI: $h(x) = e^{-x^2}$

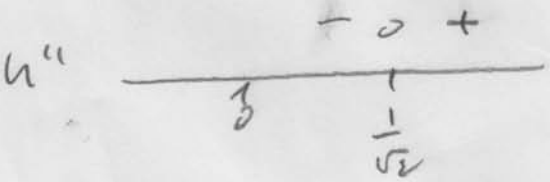
$h'(x) = -2x e^{-x^2}$ (u, v)

$h''(x) = e^{-2x}(-2) + 2x(-2x e^{-x^2})$

$= -2e^{-2x} + 4x^2 e^{-x^2}$

$= -2e^{-2x}(1-2x^2) \overset{Set}{=} 0$

$x = \frac{1}{\sqrt{2}}$



h changes from down to up at $\frac{1}{\sqrt{2}}$ ∴ has PI at $x = \frac{1}{\sqrt{2}}$ which is the same value where the rectangle has its max.

$P(w) = 2w + 2e^{-w^2}$

$P'(\frac{1}{\sqrt{2}}) = \frac{2}{\sqrt{2}} + 2e^{-\frac{1}{2}}$

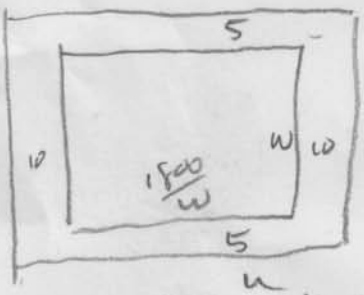
$\sqrt{2} + \frac{2}{\sqrt{e}}$

$\lim_{w\to\infty} P(w) = \lim_{w\to\infty} 2w - \frac{2}{e^{w^2}} \to +\infty$

∴ There is no max perimeter.

So, shaded item Does not have max Perimeter at that value of w and NO. That is NOT surprising!

II



$A = \left(\frac{1800}{w} + 20\right)(w+10)$

$A' = (w+10)\left(\frac{-1800}{w^2}\right) + \frac{1800}{w} + 20$

$= -\frac{1800}{w} - \frac{18000}{w^2} + \frac{1800}{w} + 20$

$\to A' = -\frac{18000}{w^2} + 20 \overset{Set}{=} 0$

$1800 = 2w^2$

$900 = w^2$

$w = 30$

$A'' = \frac{36000}{w^3} > 0$ ∴ min Area f"~~t~~

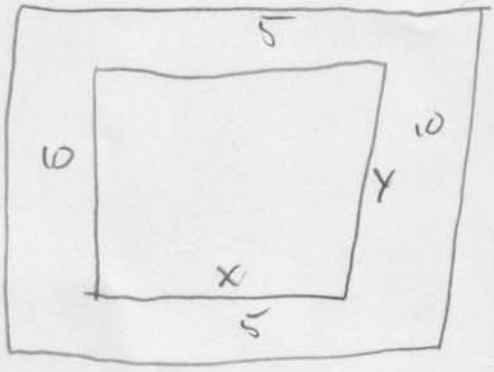
Pool Size 30 × 60

Property Size 40 × 80

Perimeter 240 FT

Fence cost @ $10/FT = $2,400.

11 min Perimeter



$XY = 1800$

$Y = \frac{1800}{X}$

$P(X) = 2(X+20) + 2\left(\frac{1800}{X} + 10\right)$

$P'(X) = 2 - \frac{3600}{X^2} \overset{\text{set}}{=} 0$

$X^2 = 1800$

$X = 9 \cdot 2 \cdot 10^2$

$= 30\sqrt{2}$

$P''(X) = \frac{7200}{X^3} > 0 \therefore$ min Perimeter.

Dimensions are

$(30\sqrt{2}+10)$ by $(30\sqrt{2}+20)$

$\$10\left[2(30\sqrt{2}+10) + 2(30\sqrt{2}+20)\right]$ is that cost.

Cost of $2,297.05

$2600 + 1200\sqrt{2}$

III

fuel $= K(\text{speed})^3$ D is distance

$\frac{100}{\text{hr}} = K(10)^3$ ← 5 pts.

so $K = \frac{1}{10\text{hr}}$

$RT = D$ so $R = x$

$T = \frac{D}{x}$ hrs

NO fctn -20

$f(x) = \frac{1}{10\text{hr}} \cdot \frac{D}{x}\text{hrs} \cdot \frac{\$}{x^3} + \frac{\$675}{\text{hr}} \cdot \frac{D}{x}\text{hrs}$

Push hrs $ + $5.675

25.27

$5^3 \cdot 3^3$

$f(x) = \frac{Dx^2}{10} + \frac{675D}{x}$

$f'(x) = \frac{D}{5}x - \frac{675D}{x^2} \overset{\text{set}}{=} 0$

$x^3 = 5.675$

$x = 15$

The ferry should travel at 15 mph to minimize costs.

f' : − 0 + (0, 15) min f'tst.

Extra

$g(x) = \frac{\ln x}{x}$, $x > 0$

$g'(x) = \frac{\frac{x}{x} - \ln x}{x^2}$

$= \frac{1 - \ln x}{x^2} \overset{\text{set}}{=} 0$ (numerator) $\ln x = 1$, $x = e$

g' : + 0 − (0, e) max at $x = e$ f'tup.

$g''(x) = \frac{x^2\left(\frac{-1}{x}\right) - (1 - \ln x)2x}{x^4}$

$\frac{-x - 2x + 2x\ln x}{x^4}$

$\frac{-3x + 2x\ln x}{x^4}$

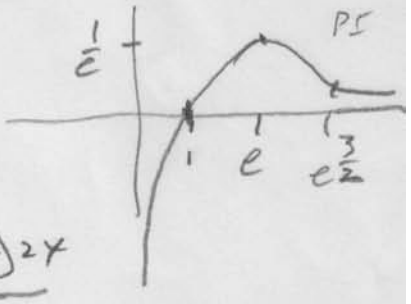
$g''(x) = \frac{-3 + 2\ln x}{x^4} \overset{\text{set}}{=} 0$

$\ln x = \frac{3}{2}$ ∴ $x = e^{\frac{3}{2}}$

g'' : − 0 + (0, $e^{\frac{3}{2}}$) dn $e^{\frac{3}{2}}$ up.

$\lim_{x \to 0^+} g(x) \to -\infty$

$\lim_{x \to \infty} g(x) \to 0$



	4	5	T
100	1	0	1
90	0	3	3
80	2	2	4
70	9	5	14
40	1	5	6
59	2	3	5

104

40

$\frac{1083}{15}$ 72.2

$\frac{1314}{18}$ 73.0

72.6

Problem II needed extra clarification I intended min area but min perimeter worked too.

Advanced Placement Calculus

Name Calc Ch6 Applic

Per _____ Date 2002

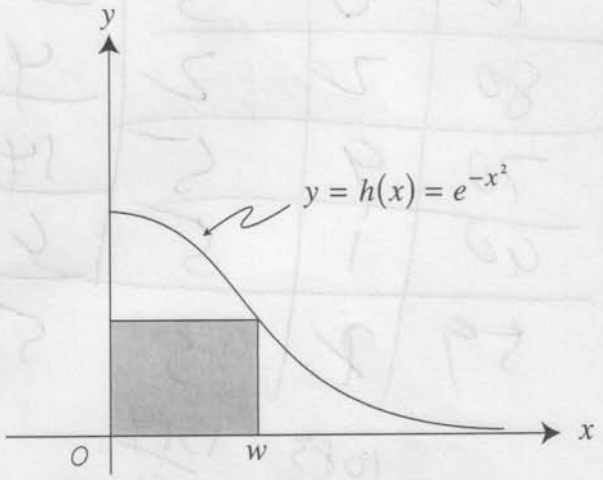
Copy original problem.

Convince *me* that **you** understand the concept*!*

No Calculators, of course.

Chapter 6 Application Exam

I Let $A(w)$ be the area of the shaded rectangle shown in the figure. Show that $A(w)$ has its maximum value when w is the x-coordinate of the point of inflection of the graph of h. Does the shaded rectangle have maximum perimeter at this same value of w? Is that a surprising result? (40 pts)



II A rectangular swimming pool is to be built with water surface area of 1800 square feet. The owner wants 5-foot wide decks along either side and 10-foot wide decks at the two ends. The owner will fence the property at a cost of \$10 per linear foot of fence. The owner wants to know the costs for fencing the smallest piece of property (in area) on which the pool can be built satisfying these conditions. Be *very* careful to have a function which is properly notated (explained and/or justified). (30 pts)

III The cost of fuel to propel a boat through the water (in dollars per hours) is proportional to the cube of the speed. A certain ferry boat uses \$100 worth of fuel per hour when cruising at 10 miles per hour. Apart from fuel, the cost of running this ferry (labor, maintenance, and so on) is \$675 per hour. At what speed should the ferry travel so as to minimize the costs? Be *very* careful to have a function which is properly notated (explained and/or justified). (30 pts)

Extra Credit ---------- 5 pts ----------

Given $g(x) = \dfrac{\ln x}{x}$ and $x > 0$. Specifically indicate any significant points on the graph of $g(x)$. Be sure all conclusions are well justified.

I Let w = # of workers $\quad \frac{1250}{4w}$ = # hrs to do job

$f(w) = w \cdot \frac{1250}{4w} \cdot 4 + 10 \cdot \frac{1250}{4w} + 8w$

(# of workers) (# hrs) ($ per hr) (Supervisor rate) · hrs + ($ per worker) · workers

$f(w) = 1250 + \frac{12500}{4w} + 8w = 1250 + \frac{3200}{w} + 8w$

$f'(w) = -\frac{3200}{w^2} + 8 \overset{\text{set}}{=} 0$

$8w^2 = 3200$

$w^2 = 400$

$w = 20$

$f''(x) = \frac{6400}{w^3} > 0 \therefore$ min

f'' test,

former should hire 20 workers.

II $f(x) = x^4 + ax^2 + b$

a) $f'(x) = 4x^3 + 2ax$

$2x(2x^2 + a)$

f has only one critical value if $a \geq 0$ and that point would be a min.

$-\ 0\ +$ on the number line at 0

b can be ANYTHING

b) $f'(x) = 2x(2x^2 + a)$

f has 3 critical values if $a < 0$

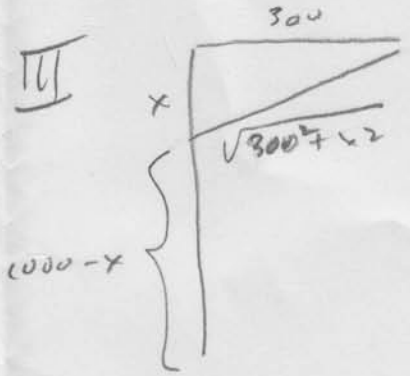
$-\ 0\ +\ 0\ -\ 0\ +$ on the number line at $-\sqrt{\frac{-a}{2}}$, 0, $\sqrt{\frac{-a}{2}}$

both the $\pm\sqrt{\frac{-a}{2}}$ are min f' test and $x = 0$ is a max f' test.

c) There cannot be 2 critical values unless Domain of f(x) is altered such as $D_f\ x \geq 0$

There could never be more than 3 because f(x) is 4th degree so at most 3 roots for the derivative.

III



$C(x) = 2\sqrt{300^2 + x^2} + 1000 - x$

$C'(x) = \frac{2 \cdot 2x}{2\sqrt{300^2 + x^2}} - 1 \overset{\text{set}}{=} 0$

$2x = \sqrt{300^2 + x^2}$

$4x^2 = 300^2 + x^2$

$3x^2 = 300 \cdot 300^2$

$x = 100\sqrt{3}$

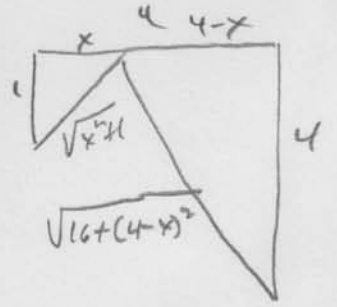
Point P is $100\sqrt{3}$ miles South of Point C.

C' $-\ 0\ +$ on the number line at 0, $100\sqrt{3}$

$C'(0) = 0 - 1 < 0$ ↓, min

Cont f' test,

IV



$$f(x) = \sqrt{16+(4-x)^2} + \sqrt{x^2+1}$$

$$f'(x) = \frac{-2(4-x)}{2\sqrt{16+(4-x)^2}} + \frac{2x}{2\sqrt{x^2+1}} \overset{\text{set}}{=} 0$$

$$\frac{4-x}{\sqrt{16+(4-x)^2}} = \frac{x}{\sqrt{x^2+1}}$$

$$\frac{16-8x+x^2}{16+16-8x+x^2} = \frac{x^2}{x^2+1}$$

$$16x^2 - 8x^3 + x^4 + 16 - 8x + x^2$$

$$x^4 - 8x^3 + 17x^2 - 8x + 16 = 32x^2 - 8x^3 + x^4$$

$$15x^2 + 8x - 16 = 0$$

$$(5x-4)(3x+4) = 0$$

Reject

$$5x = 4$$

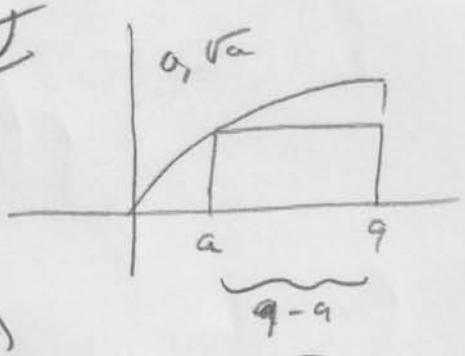
$$x = \frac{4}{5}$$

$$f'(0) = \frac{-4}{\sqrt{32}} + 0 < 0$$

f': $-$ | 0 | $+$ (at 0 and $\frac{4}{5}$)

min cost when Pump station is .8 mile east of Town 1.

V



a)

$$A(a) = \sqrt{a}(9-a)$$

$$A'(a) = \frac{(9-a)}{2\sqrt{a}} - \sqrt{a} \overset{\text{set}}{=} 0$$

$$9-a = 2a$$

$$9 = 3a$$

$$3 = a$$

A': $+$ | 0 | $-$ (at 0 and 3)

$$A'(1) = \frac{8}{2} - 1 > 0$$

min area if $a = 9$ is nonrectangle.

max area when $a = 3$ $f'(x)$

b)

$$P(x) = 2(9-a) + 2\sqrt{a}$$

$$P'(x) = -2 + \frac{2}{2\sqrt{a}} \overset{\text{set}}{=} 0$$

$$\frac{1}{\sqrt{a}} = 2$$

$$\sqrt{a} = \frac{1}{2}$$

$$a = \frac{1}{4}$$

$$P'(1) = -2 + 1 < 0$$

P': $+$ | 0 | $-$ (at 0, $\frac{1}{4}$, 1)

Like part a min Perimeter if $a = 9$ if NOT.

Max perimeter at $x = \frac{1}{4}$

1/14/03

Bob Marcus 73
Kristen Jones 58
Eamonn Gahon 53
Rachel Heinz 52
Elizabeth Langley 25

Advanced Placement Calculus

Name Ch 6 Top Mu

Per _____ Date 2002

Copy original problem.

Convince *me* that **you** understand the concept*!*

No Calculators, of course. Each section 20 pts

Chapter 6 Applications Make–Up Exam

I A farmer wants to hire workers to pick 1,280 bushels of beans. Each worker can pick 4 bu/hr and is paid \$1.00/bu. The farmer must also pay a supervisor \$10 per hour while the picking is in progress, and he has miscellaneous additional expenses of \$8 for each worker hired. How many workers should he hire in order to minimize the total cost? What will then be the cost per bushel picked?

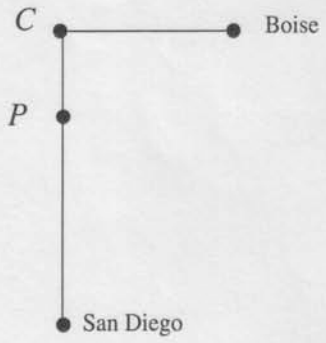
II Given: $f(x) = x^4 + ax^2 + b$

a) Under what conditions on a and b does $f(x)$ have exactly one critical point? What is the one critical point, and is it a local maximum, a local minimum, or neither?

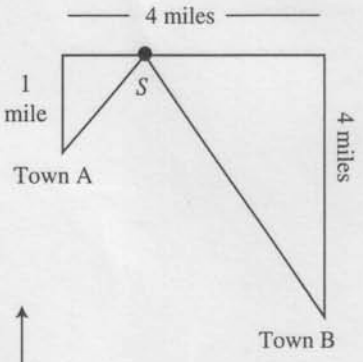
b) Under what conditions on a and b does $f(x)$ have exactly three critical points? What are they? Which are local maxima and which are local minima?

c) Is it ever possible for $f(x)$ to have two critical points? More than three critical points? Give an explanation in each case.

III Boise, Idaho is about 300 miles inland from the nearest point on the Pacific coast. San Diego is about 1000 miles south of that point down the coast. Assuming the coast is a straight line going north-south, C, is the point along the coast directly west of Boise. It costs 2 cents per mile to transport a ton of potatoes by truck and 1 cent per mile to transport them by ship. The Idaho Potato Company wants to find the point, P, on the Pacific coast to which it should truck its potatoes before loading them aboard a cargo ship in order to minimize the total cost of transporting them from Boise to San Diego. Find that point.



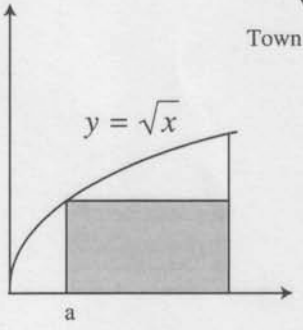
IV On the same side of a straight river are two towns and the townspeople want to build a pumping station, S. The pumping station is to be at the river's edge with pipes extending straight to the two towns. Where should the pumping station be located to minimize the total length of pipe?



V The figure shows the curves: $y = \sqrt{x}$; $x = 9$; $y = 0$ and a rectangle with its sides parallel to the axes and its left end at $x = a$.

a) Find the dimensions of the rectangle which is an extreme value for the area. Which is it? (max or min?)

b) Find the dimensions of the rectangle which is an extreme value for the perimeter. Which is it? (max or min?)



A P Calculus Chapter 6 2002

We *love*

Applications

Ch 6 Mech Exam

1 R/C pg 216 - 222
(skip example 5)
2 Pg 223 - 224 1 – 13
3 Pg 224 15, 17

11/22

Turn in Hw #1, 2

During Vacation:

4 Pg 225 19, 20
Write a very detailed argument explaining your reasoning for problems 19 and 20.

11/ 25

5 Pg 225 22, 27 - 31
Extend problem 22 with a Generalization by replacing the equation of the linewith: $y = mx + b$.

11/26

Thanksgiving Vacation

6 Pg 225 - 226
32, 33, 35
Use a calculator to solve the quadradic equation in problem 32

11/27

7 and #8
Read pgs 226 - 231
Then write a lecture covering that material.

#9
Pg 231 - 232 1 - 21, 23

11/28

10, 11
Rent, Borrow or whatever then **VIEW** the movie **The Legend of Bagger Vance**
Explain what happened to Jack Lemmon's character on the golf course in the very beginning of the movie and how it ties to the closing of movie.
One paragraph maximum

#12, 13, 14, 15, 16, **"The Continuing Legend of Bagger Vance"**

12 / 2

12 / 3

12 / 4

12 / 5

17
GQ C 6 pg 242 17, 18
GQ ch 1-6 Comp pg 243 10
GQ ch 1-6 Conc pg 244 7A

12 / 6

18
Pg 245 - 246
27 - 29, 35ab, 40, 41, 43, 51

12 / 9

18 Pg 246 - 247
56, 57, 58, 60, 61, 64, 65
19
Pg 247 78 - 80

12/ 10

Put Last Year's Exam On The Chalkboard.

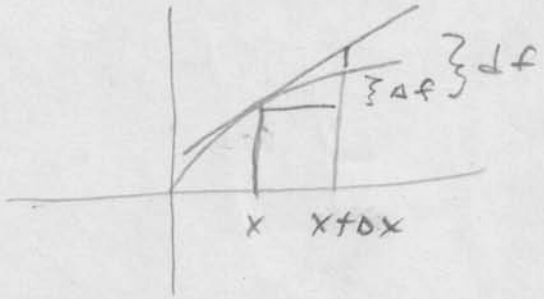
Come to class Prepared!

12/ 11

CALCULUS EXAM Chapter 6 "APPLICATIONS"

12/ 12

I df Approximation change
Δf Actual change



$f(x+\Delta x) = f(x) + \Delta f$

$f(x+\Delta x) \approx f(x) + f'(x)\,df$

df is greater than Δf when f is concave down

II $\cos 29°$ $\quad \Delta x = -\frac{\pi}{180}$

$f(x) = \cos x \quad x = \frac{\pi}{6}$

$f\left(\frac{\pi}{6} - \frac{\pi}{180}\right)$

$f(x+\Delta x) \approx f(x) + f'(x)\,\Delta x$

$\approx \cos\left(\frac{\pi}{6}\right) - \left(\sin\frac{\pi}{6}\right)\frac{-\pi}{180}$

$\approx \frac{\sqrt{3}}{2} + \frac{1}{2}\left(-\frac{\pi}{180}\right)$

$\approx \frac{1.75 + .0175}{2}$

≈ 1.7675

$\approx .88375$

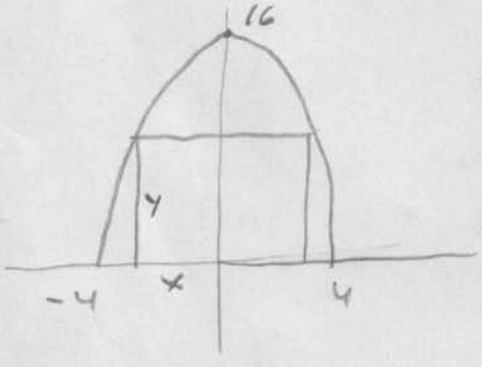
$f(x) = \sqrt{x}$ $\quad f(3) \approx f(4) + \frac{1}{2\cdot 2}(-1)$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\quad 2 - .25$

$x = 4$ $\quad 1.75$

$dx = -1$

III



$y = 16 - x^2$

$A = bh$

$A = 2xy$

$A(x) = 2x(16 - x^2)$

$= 32x - 2x^3$

$A' = 32 - 6x^2 \overset{set}{=} 0$

$3x^2 = 16$

$x = \frac{4\sqrt{3}}{3}$

$A'' = -12x < 0 \therefore$ max

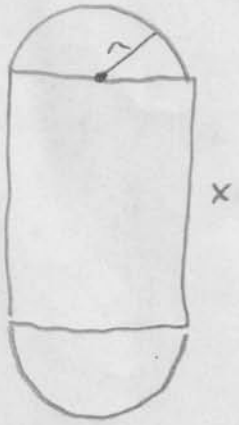
f'' test.

base $\frac{8\sqrt{3}}{3}$

height $\frac{32}{3}$.

48 $\frac{16}{3}$

IV



max Rectangle Per of Track 440

circle + rectangle = 440

$2\pi r + 2x = 440$

$x = 220 - \pi r$

$A = bh$

$A(r) = 2r(220 - \pi r)$

$A(r) = 440r - 2\pi r^2$

$A' = 440 - 4\pi r \overset{set}{=} 0$

$r = \frac{110}{\pi}$

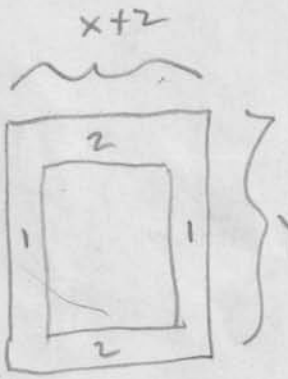
$220 - 110 = 110$

dimension:

$2r \times x$

$\frac{220}{\pi} \times 110$

J



$xy = 30$

$A = (x+2)(y+4)$

$A(x) = (x+2)\left(\frac{30}{x} + 4\right)$

$A(x) = 30 + \frac{60}{x} + 4x + 8$

$A' = -\frac{60}{x^2} + 4 \overset{set}{=} 0$

$x^2 = 15$

$x = \sqrt{15}$

$y = \frac{30}{\sqrt{15}} = 2\sqrt{15}$

dimensions: $2\sqrt{15} + 4 \times \sqrt{15} + 2$

Extra

$y = \sin^2\theta \sin 2\theta$

$y' = 2\sin 2\theta \sin\theta\cos\theta + 2\sin^2\theta \cos 2\theta$

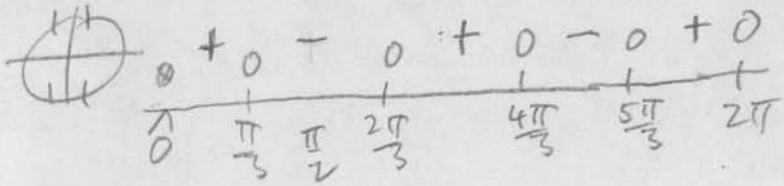
$= 4\sin^2\theta\cos^2\theta + 2\sin^2\theta(\cos^2\theta - \sin^2\theta)$

$= 2\sin^2\theta(3\cos^2\theta - \sin^2\theta)$

$= 2\sin^2\theta(4\cos^2\theta - 1)$

$= 2\sin^2\theta(2\cos\theta - 1)(2\cos\theta + 1)$

$A'' = \frac{120}{x^3} > 0 \therefore$ min f'' Crit.



increasing for $0 < x < \frac{\pi}{3}$ or $\frac{2\pi}{3} < x < \frac{4\pi}{3}$ or $\frac{5\pi}{3} < x < 2\pi$

$\frac{2226}{26}$ 85.62

100	2
90	12
80	4
70	4
60	3
59	2

Apparently this is exactly the Program length!

Advanced Placement Calculus AB Name Calc Ch 6 App 1990

Copy the original problem.
Convince *me* that **you** understand the concept. Per _____ Date _____

CHAPTER #6 APPLICATIONS

I Explain the difference(s) between df and Δf. Under what conditions will df be greater?

(10 pts)

II In calculus, when we work with trigonometry functions, we use radians because they have "cleaner" derivatives. For example, the derivative of Sin x is Cos x, if x is measured in radians. If, on the other hand, x is measured in degrees then the derivative of Sin x is $\frac{\pi}{180}$ Cos x. The $\frac{\pi}{180}$ compensates for the differences between degrees and radians. $\frac{\pi}{180} = 0.0175$. It could be considered the radian equlivalent of one degree.

Compute (without using a calculator) the approximate value for Cos 29° using differentials.
Your final answer is to be decimal and contain NO radicals

(20 pts)

III A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

(25 pts)

IV A playing field is to be built in the shape of a rectangle plus a semicicular area at each end. A 440-yard race track is to form the permeter of the field. Find the *dimensions of the rectangluar part* of the field **if the rectangular part** is to have as large an area as possible. (20 pts)

V The pages of a book are each to contain thirty square inches of print, with two inch margins at the top and bottom of each page and one inch side margins. What are the dimensions of a page having minimum area? (hint: final dimensions of the page *does* contain radicals.) (20 pts)

Extra Credit ═══ 5 pts ═══

Given: $y = \text{Sin}^2 x \text{ Sin } 2x$ with $x \in [0, 2\pi]$. Define the sub-intervals where y is increasing.

I

A) $f(x) = e^{-x^2}$

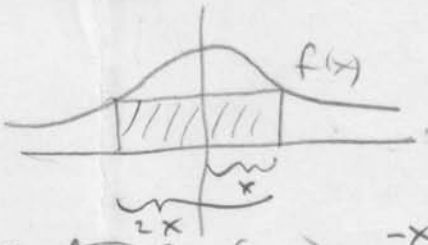
$f(-x) = e^{-(-x)^2}$

$= e^{-x^2}$

$= f(x)$

$\therefore f$ is even

B)



Rectangle $(2x)e^{-x^2} = A$rea

$f(x) = \overset{u}{2x}\overset{v}{e^{-x^2}}$

$f'(x) = 2e^{-x^2} - 4x^2e^{-x}$

$2e^{-x^2}(1-2x^2) \overset{Set}{=} 0$

Never zero

$x^2 = \frac{1}{2}$

$x = \frac{\sqrt{2}}{2}$

max by f' test.

$f' =$ + 0 − on 0, $\frac{\sqrt{2}}{2}$

Dimensions: $\sqrt{2} \times e^{-\frac{1}{2}}$

II

$V(r) = \pi r^2 \sqrt{1-4r^2}$ (u, v)

$V' = 2\pi r\sqrt{1-4r^2} + \frac{\pi r^2(-8r)}{2\sqrt{1-4r^2}}$ (4)

$= \frac{2\pi r(1-4r^2) - 4r^3}{\sqrt{1-4r^2}} \overset{Set}{=} 0$

$2\pi r - 8\pi r^3 - 4\pi r^3$

$2\pi r - 12\pi r^3$

$2\pi r(1-6r^2) = 0$

$r = \frac{\sqrt{6}}{6}$

$h = \sqrt{1-\frac{4}{6}}$

$h = \sqrt{\frac{1}{3}}$

$h = \frac{\sqrt{3}}{3}$

V' · + 0 − on 0, $\frac{\sqrt{6}}{6}$

max V' test.

Dimensions: height $\frac{\sqrt{3}}{3}$, radius $\frac{\sqrt{6}}{6}$

III

A) $f(x) = x^2$ $x = 4$

$f'(x) = 2x$ $\Delta x = .02$

$f(4.02) \approx x^2 + f'(x)\Delta x$

$4^2 + 2(4)\frac{2}{100}$

16.16

B) $f(x) = \sqrt{x}$ $x = 25.16$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\Delta x = .16$

$f(25.16) \approx \sqrt{25} + \frac{1}{2.5}\cdot\frac{16}{100}$

$5 + \frac{16}{1000}$

≈ 5.016

IV

$f(\tau) = Ae^{-k\tau}$

$f(\tau) = 6e^{-k\tau}$

$4 = 6e^{-k}$

$e^{-k} = \frac{2}{3}$

$\therefore f(\tau) = 6\left(\frac{2}{3}\right)^{\tau}$

$f(10) = 6\left(\frac{2}{3}\right)^{10}$

Extra ... S = speed of train.

$f = kS^2$

$50 = k(20)^2$

$50 = 400k$

$k = \frac{1}{8}$

fuel costs/hr $= \frac{1}{8}S^2$

other charges/hr. $200

$R \cdot \tau = D$

$\tau = \frac{D}{R} \therefore \frac{500}{S}$ hrs.

$f(S) = \frac{1}{8}S^2\left(\frac{500}{S}\right) + \frac{500}{S}(200)$

$= 500\left(\frac{1}{8}S + \frac{200}{S}\right)$

$f' = 500\left(\frac{1}{8} - \frac{200}{S^2}\right) \overset{\text{set}}{=} 0$

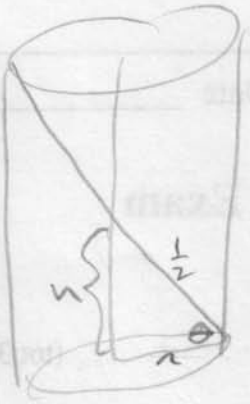
$S^2 = 1600$

$S = 40$

$f'' = 500\left(\frac{400}{S^3}\right) > 0$

$\therefore$ min at $S = 40$

f'' test.



$\frac{r}{\frac{1}{2}} = \cos\theta$

$\frac{1}{2}\cos\theta = r$

$\frac{h}{\frac{1}{2}} = \sin\theta$

$h = \frac{1}{2}\sin\theta$

$h = \frac{\sqrt{3}}{6}$

$V = \pi r^2 h$

$V = \frac{\pi}{4}\cos^2\theta \cdot \frac{1}{2}\sin\theta$

$= \frac{\pi}{8}\cos^2\theta \sin\theta$ $\quad 1-\sin^2\theta$

$= \frac{\pi}{8}\left[-2\sin^3\theta\cos\theta + \cos^3\theta\right]$

$\frac{\pi}{8}\cos\theta(-2\sin^2\theta + \cos^2\theta)$

$V = \frac{\pi}{8}(\sin\theta - \sin^3\theta)$

$V' = \frac{\pi}{8}\cos\theta - 3\sin^2\theta\cos\theta$

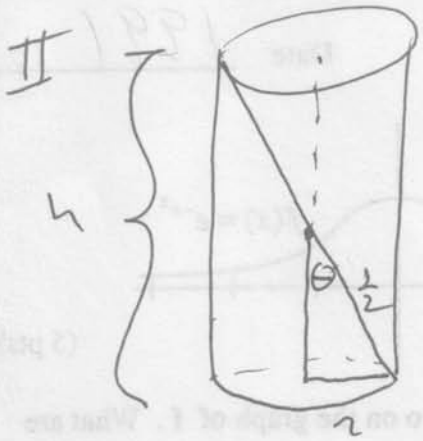
$\frac{\pi}{8}\cos\theta(1-3\sin^2\theta) \overset{SA}{=} 0$

$3\sin^2\theta = 1$

$\sin\theta = \frac{1}{\sqrt{3}}$

NO

II



$\frac{r}{\frac{1}{2}} = \sin\theta$ $\qquad$ $\frac{\frac{h}{2}}{\frac{1}{2}} = \cos\theta$.

$V = \frac{1}{3}\pi r^2 h$

$= \pi(4\sin^2\theta)\cos\theta$

Looks promising. etc.

II

A) ~~do~~ in terms of r

B) do in terms of θ.

C) are answers the same? etc.

100	1
90	5
80	1
70	6
60	0
59	3

103

30

There not included. He doesn't speak English well enough to read the questions I think.

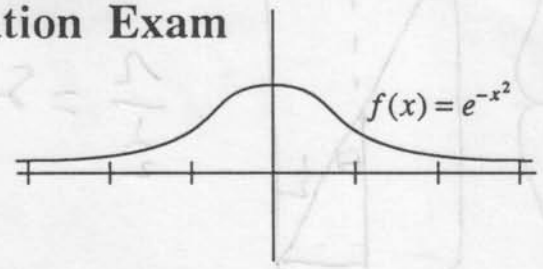
$\frac{1232}{15} = 82.13$

A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!

Name Calc Ch 6 rech

Per _____ Date 1991

Chapter 6 Application Exam



I Given $f(x) = e^{-x^2}$. See figure.

A) Prove f is odd, even or neither. (5 pts)

B) A rectangle, R, has two of its verticies on the x-axis and the other two on the graph of f. What are the dimensions of R which has the largest area? (20 pts)

II In the seventeenth century, the famous astronomer Kepler became interested in the measurement of wine casks. He proposed the following problem:

> If a wine barrel in the shape of a right circular cylinder has a diagonal length of one meter, what are the dimensions of the barrel which has the maximum volume?

Knowing the volume of a right circular cylinder is given by: $v = \pi r^2 h$, solve Kepler's problem. (25 pts)

III Differentials (25 pts tot)

A) Use differentials to approximate: $(4.02)^2$

B) Use differentials to find the approximate length of the hypotenuse of a right triangle whose legs have lengths 4.02, and 3.

IV A certain chemical substance decomposes at a rate proportional to the amount present. If, initially, there were six grams of the substance, and two grams decomposed in one minute, how many grams are left after ten minutes? (25 pts)

Extra Credit **5 pts**

The fuel costs for operating a train is proportional to the square of the speed, and is \$50 per hour when the speed is 20 mph. Other charges (per hour) such as labor (for example) are estimated to be \$200. What should be the speed for a 500 mile trip in order to minimize the total cost?

I

a) $600 - 12(x-24)$

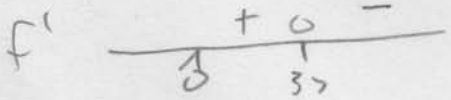
b) $f(x) = x(600 - 12(x-24))$

c) $= 600x - 12x^2 + 288x$

$f(x) = 888x - 12x^2$

$f'(x) = 888 - 24x \overset{\text{set}}{=} 0$

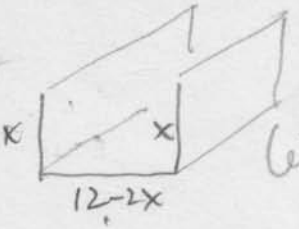
$x = \frac{888}{24} = 37$



max f' test.

Plant 37 trees per acre

b)



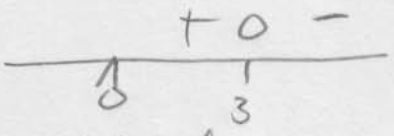
Let l = length

$V = x(12-2x)l$

$= (12x - 2x^2)l$

$V' = 12 - 4x \overset{\text{set}}{=} 0$

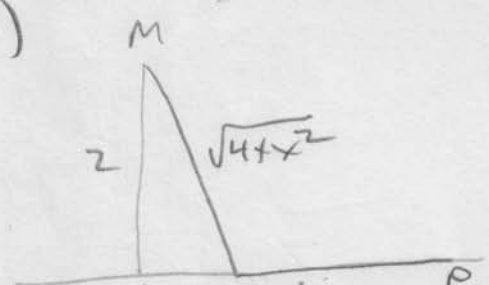
$x = 3$



max f' test.

fold up 3" on each side.

c)



$RT = D$

$T = \frac{D}{R}$

$T(x) = \frac{\sqrt{4+x^2}}{3} + \frac{6-x}{5}$.

$T' = \frac{x}{3\sqrt{4+x^2}} - \frac{1}{5} \overset{\text{set}}{=} 0$

$5x = 3\sqrt{4+x^2}$

$25x^2 = 9(4+x^2)$

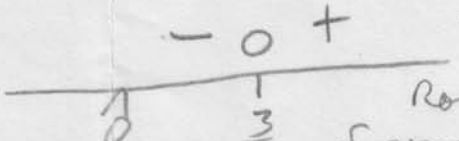
$16x^2 = 36$

$x = \pm \frac{3}{2}$ reject $-\frac{3}{2}$

$T'(1) = \frac{1}{3\sqrt{5}} - \frac{1}{5}$

$\frac{\sqrt{5}}{15} - \frac{3}{15} < 0$

min f' test.



Row to a point $4\frac{1}{2}$ miles from destination and run from there.

II

a) $N(t) = Ae^{kt}$

$N'(t) = Ake^{kt}$

$= kAe^{kt}$

$= kN(t)$

b) $2A \overset{\text{Set}}{=} Ae^{20k}$

$2 = e^{20k}$

$\ln 2 = 20k \ln e$

$k = \frac{\ln 2}{20}$

$k = .0347$

c) $3A = Ae^{kt}$

$3 = e^{kt}$

$\ln 3 = kt$

$\frac{\ln 3}{.0347} = t$

$t = 31.7$ years

III

a) $y = x^3 - 3x^2 + 2x - 1$

$y' = 3x^2 - 6x + 2 \qquad y'' = 6x - 6$

$x = 2 \qquad \Delta x = -.002$

$f(2 - .002) = f(2) + f'(2)\,\Delta x$

	1	−3	2	−1
2	1	−1	0	$-1 = f(2)$
2	1	1	2	$= f'(2)$

$-1 + 2(-.002)$

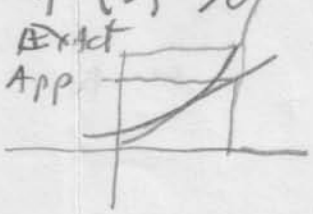
$-1 - .004$

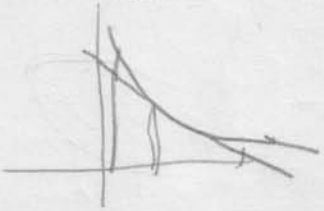
-1.004

$y''(2) > 0$ concave up.

$y'(2) > 0$ increasing.

df is less than Δf.





Extra.

$f(r) = c^2 r + \frac{A}{r}$

$f' = c^2 - \frac{A}{r^2} \overset{\text{Set}}{=} 0$

$\frac{A}{r^2} = c^2$

$r^2 = \frac{A}{c^2}$

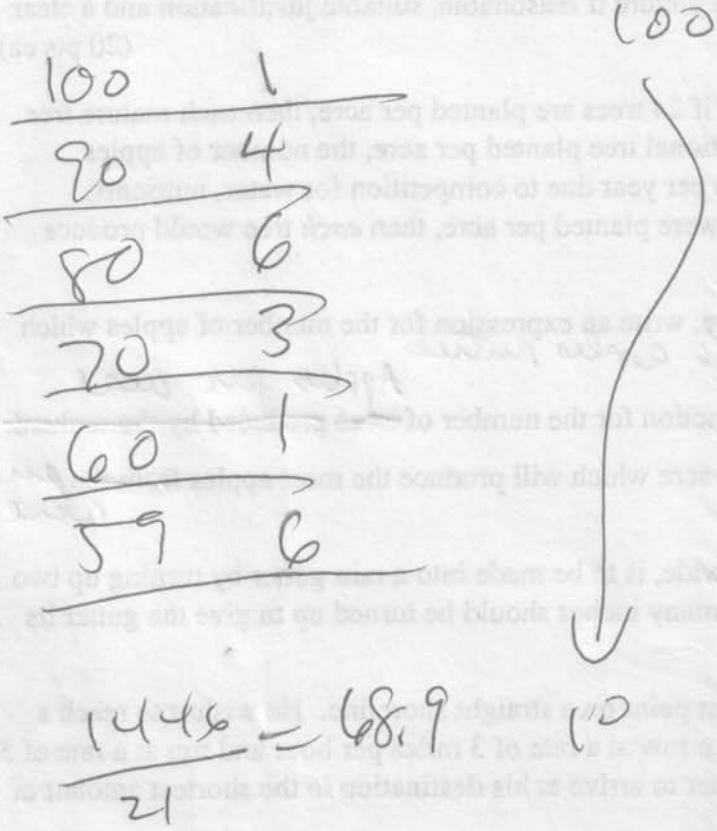
$r = \frac{\sqrt{A}}{c}$

f' : − 0 +

0 | $\frac{\sqrt{A}}{c}$

min f' test.

Excellent, easy exam —
Lousey students!



100

$\frac{1446}{21} = 68.9$ (10)

Advanced Placement Calculus Name 92 Calc

Copy original problem. Per _____ Date C6 Applic

Convince *me* that **you** understand the concept!

Chapter 6 Applications Exam

I Solve each of the following. Be sure to include a picture if reasonable, suitable justification and a clear but brief explanation of your method. (20 pts ea)

a) The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will yield 600 apples per year. For each additional tree planted per acre, the number of apples produced by each tree decreases by 12 apples per year due to competition for water, nutrients, sunlight, etc. (Thus, for example, if 25 trees were planted per acre, then *each* tree would produce $600 - 12(25 - 24) = 588$ apples per year.)

1) If x is the number of trees per acre, write an expression for the number of apples which will be produced by each tree. # of apples per tree Apples per acre.

2) Continuing from above, write a function for the number of ~~trees produced by the orchard.~~

3) Determine the number of trees per acre which will produce the most apples ~~from the orchard.~~ per acre.

b) A long rectangular sheet of metal, 12 inches wide, is to be made into a rain gutter by turning up two equal sides at right angles to the sheet. How many inches should be turned up to give the gutter its greatest volume?

c) A man in a rowboat is 2 miles from the nearest point on a straight shoreline. He wishes to reach a point 6 miles further down the shore. If he can row at a rate of 3 miles per hour and run at a rate of 5 miles per hour, how should he proceed tin order to arrive at his destination in the shortest amount of time?

II $N(t)$ is the population in a certain country in terms of years. The population of the country doubles every twenty years. The rate of increase in population is governed by the equation: $\frac{dN}{dt} = kN$ where k is a constant and t is the time in years. (20 pts tot)

a) Show that $N(t) = Ae^{kt}$ satisfies the growth equation given above.

b) Find the growth constant, k for this country (Use a calculator — 4 decimal places)

c) About how many years will it take for the population to be three times what it is now?

III Differentials (20 pts tot)

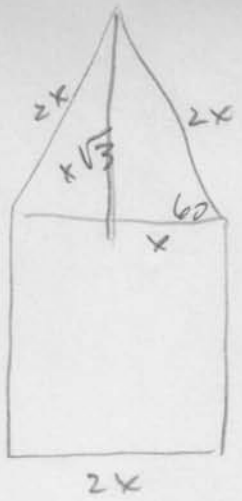
a) Find an approximation to the value of $y = x^3 - 3x^2 + 2x - 1$ when $x = 1.998$.

b) Is your approximation greater or less than the actual amount? Why?

Extra Credit ----------5 pts ----------

The power loss in watts per mile in an electric conductor is: $C^2r + \frac{A}{r}$ where C is the current in amps, r is the resistance in ohms per mile. For a given current, what is the resistance required for minimum loss of power per mile?

I



$6x+2y=200$

$3x+y=100$

$y=100-3x$

Area $\triangle$ $\frac{1}{2}\cdot 2x\cdot x\sqrt{3}$

$= x^2\sqrt{3}$

Area $\square = 2x\,y$

$= 200x-6x^2$

$f(x)= x^2\sqrt{3}+200x-6x^2$

$f'(x)= 2x\sqrt{3}+200-12x \overset{\text{set}}{=} 0$

$2x(\sqrt{3}-6) = -200$

$2x = \frac{200}{6-\sqrt{3}}$

$f''(x)= 2\sqrt{3}-12<0$ since $2\cdot 2 < 2.6$ and $2>\sqrt{3}$

So we have max f'' test.

II

let w = weeks to wait

$10000-100w$ = weight on sale day

$160+10w$ = New price on Sale day

$60000w$ = Total dollars for w weeks

$f(w) = (10000-100w)(160-10w) - 600.00w$

$f(w) = 1000(100-w)(16-w) - 1600(60$

$= 1000[1600+84w+w^2-60w]$

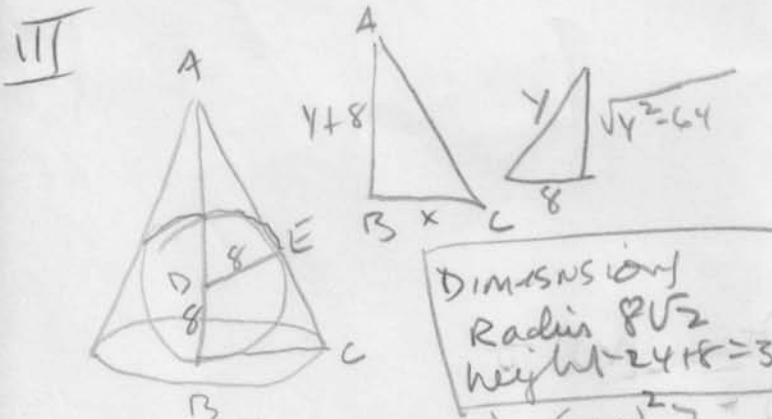
$= 1000[1600+24w+w^2]$

$f' = 1000[24+2w] \overset{\text{set}}{=} 0$

$w = 12$

$f'' = 2000 > 0$, max at $w=12$, f'' test

III



DIMENSIONS
Radius $8\sqrt{2}$
height $=24+8=32$

$\frac{x}{8} = \frac{Y+8}{\sqrt{Y^2-64}}$

$x = \frac{8(Y+8)}{\sqrt{Y^2-64}}$

$V = \frac{1}{3}\pi x^2 h$

$V = \frac{1}{3}\frac{\pi 64(Y+8)^2}{Y^2-64}\cdot Y+8$

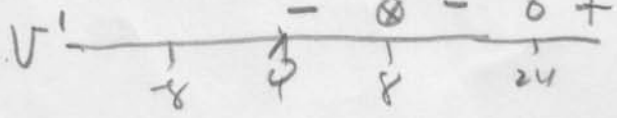
$V = \frac{64\pi(Y+8)^2}{3(Y-8)}$

$V' = \frac{64\pi}{3}\left[\frac{(Y-8)2(Y+8)-(Y+8)^2}{(Y-8)^2}\right]$

$= \frac{64\pi}{3}\frac{Y+8}{(Y-8)^2}\left[2(Y-8)-(Y+8)\right]$

$= \frac{64\pi}{3}\frac{Y+8}{(Y-8)^2}(Y-24) \overset{\text{set}}{=} 0$

$Y \in \{-8, 24\}$



min at $y=24$ f' test.

$\frac{x}{8} = \frac{32}{\sqrt{8^2\cdot 3^2 - 8^2\cdot 8^2}} = \frac{32}{8\sqrt{3^2-1}}$

$x = \frac{32}{\sqrt{8}} = \frac{16}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$ = Radius

IV

$y = 4x^3 - 6x^2 + 3x + 5$

$\frac{dy}{dx} = 12x^2 - 12x + 3$ = change of y value

$\frac{d(\frac{dy}{dx})}{dx} = \frac{d^2y}{dx^2} = 24x - 12 \overset{set}{=} 12x^2 - 12x + 3$

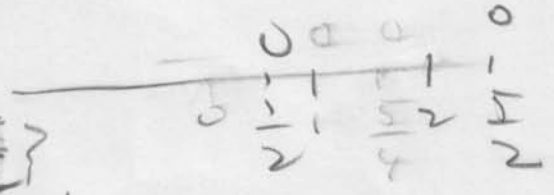
change of slope of y

$12x^2 - 36x + 15 = 0$

$4x^2 - 12x + 5 = 0$

$(2x - 5)(2x - 1) = 0$

Both items change when $x \in \{\frac{1}{2}, \frac{5}{2}\}$.



Extra

$y = 2x^2 - 6x + \frac{5}{2}$

$2(x^2 - 3x + \frac{9}{4}) + \frac{5}{2} - \frac{9}{2}$

$= 2(x - \frac{3}{2})^2 - 2$

vertex $(\frac{3}{2}, -2)$

$\frac{1}{4c} = 2$

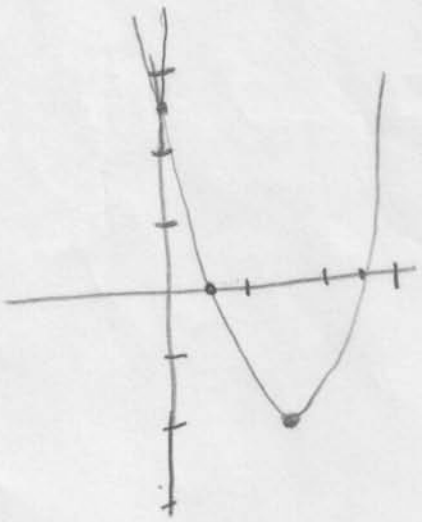
$c = \frac{1}{8}$

focus $\frac{3}{2}, -2 + \frac{1}{8}$

$(\frac{3}{2}, -\frac{15}{8})$

directrix $y = 2 - \frac{1}{8}$

$y = -\frac{17}{8}$



$2x^2 - 6x + \frac{5}{2} \overset{set}{=} 0$

$4x^2 - 12x + 5 = 0$

$(2x - 5)(2x - 1) =$

$x \in \{\frac{5}{2}, \frac{1}{2}\}$ x intercepts

y intercept when $x = 0 \therefore (0, \frac{5}{2})$

100	1
90	3
80	6
70	8
60	2
	1

100

42

$$\frac{1666}{21} = 79.3$$

Very easy exam ....
No changes except
perhaps focus on
students!

Advanced Placement Calculus

Name Calc C6 AP

Per _____ Date 93

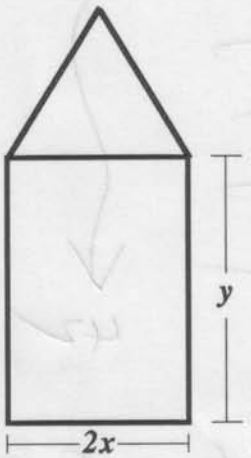
Copy original problem.

Convince *me* that **you** understand the concept!

Chapter 6 Applications Exam

Each problem carries 25 points.

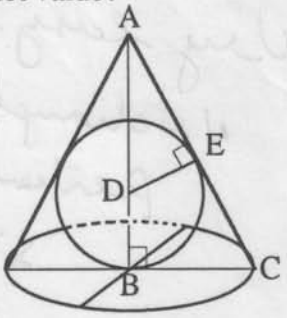
I A window is formed by placing an equilateral triangle is on a rectangle as shown on the right. The *total* perimeter of the window (the distance around the exterior of the figure) is exactly two hundred feet. For the window to admit the maximum light (i.e. have the largest surface area) show that the base of the rectangular portion of the window is: $200\left(6-\sqrt{3}\right)^{-1}$



II Ten thousand pounds of beef in cold storage are worth one dollar and sixty cents per pound, wholesale. If the price increases steadily at ten cents per pound per week while the beef loses a hundred pounds a week in weight (due to shrinkage) and the storage costs are six hundred dollars per week, how many weeks should the beef be held before selling for the greatest net value?

III Find the dimensions of the right circular cone of minimum volume V that can be circumscribed about a sphere of radius eight inches as shown in the diagram at the right. DE = DB = 8.
Hint: Let BC = x and AD = y.
Note: $\Delta AED \approx \Delta ABC$ because both are right triangles and they have a common angle.

$V_{cone} = \frac{1}{3}\pi r^2 h.$



IV Along the graph of the equation $y = 4x^3 - 6x^2 + 3x + 5$ both the ordinate y and the slope (of the tangent) m change, but generally at different rates. Find the point (or points), if any, where the ordinate and the slope are changing at the same rate.

Extra Credit ---------- 5 pts ----------

Given: $y = 2x^2 - 6x + \frac{5}{2}$. Determine the equation of the directorix and the coordinates of the focus. Sketch. List *all* significant items (such as intercepts).

I

a) $P = .4Nx^2$ $\quad P'dp$ $\quad P'(x)\,dP$

$N = 4$ $\quad x = 3\frac{1}{4} = \frac{13}{4}$

$P = 1.6x^2$ $\quad \Delta x = .015 = \frac{15}{1000}$

$P' = 3.2x$

$\frac{32}{10}x$

$\frac{16}{5}x$

$4 \cdot \frac{16}{5} \cdot \frac{13}{4} \cdot \frac{15}{1000}$ $\quad \frac{12}{13}$ / $\frac{36}{12}$

$\frac{156}{1000} = .156$

b) $\tan 46^\circ$

$\tan(45^\circ + 1^\circ)$

$\tan\left(\frac{\pi}{4} + .0175\right) \approx \tan\frac{\pi}{4} + \sec^2\frac{\pi}{4}(.0175)$

$1 + 2(.0175)$

$1 + .035 = 1.035$

II

$f(0) = 100$ $\quad f(t) = Ae^{kt}$

$f(3.5) = 250$ $\quad f(t) = 100e^{kt}$

$f(3.5) = 250 \overset{\text{Sol}}{=} 100e^{3.5k}$

$2.5 = e^{3.5k}$

$\ln 2.5 = 3.5k$

$.91629$

$.2618 = k$

$e^k = 1.2993$

$f(t) = 100(1.3)^t$

$400 = 100(1.3)^t$

$4 = (1.3)^t$

$\frac{\ln 4}{\ln(1.3)} = t$

$t = 5.3$

$5 \text{ hrs } \frac{3}{10} = \frac{18}{60}$

400 Grams at 6:18 pm.

III 1) Let x = # Sold

Production cost $= 50x + 20000$
(Raw materials, Rent)

Profit: $P(x) = \underbrace{(200 - .01x)}_{\text{Sales Price}} \underbrace{x}_{\text{Num of Items}} - (50x + 20000)$ ← less costs

max Production selling 7500 Items.

$200 - .01(7500)$

$200 - 75$

\$1.25 Sales Price

$P(7500) = (125)(7500) - 50(7500 + 400)$

$937500 - 375000$

$= \$562500$

2) $P(x) = 200x - .01x^2 - (50x + 20000)$

$P'(x) = 200 - .02x - 50 \overset{\text{Set}}{=} 0$

$.02x = 150$

$\frac{1}{50}x = 150$

$x = 7500$

$P'' = -.02 < 0 \therefore$ max f'' test.

3) Costs change ← The tax

$50x + 20000 + 10x$

$60x + 20000$

$P(x) = 200x - .01x^2 - 60x + 20000$

$P' = 200 - .02x - 60 \overset{\text{Set}}{=} 0$

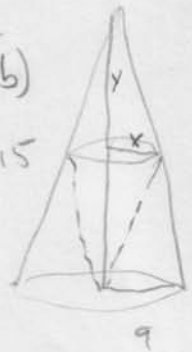
$.02x = 140$

$x = 7000$

Price: $(200 - (.01)7000)$

New Price \$1.30 Pays 5¢ on to customer.

III b)



$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi x^2 (15 - \frac{5}{3}x)$

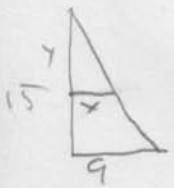
$V(x) = 5\pi x^2 - \frac{5}{9}\pi x^3$

$V' = 10\pi x - \frac{5}{3}\pi x^2 \overset{\text{Set}}{=} 0$

$5\pi x\left(2 - \frac{1}{3}x\right) = 0$

Reject $\frac{1}{3}x = 2$

$x = 6$



$\frac{y}{15} = \frac{x}{9}$

$y = \frac{5}{3}x$

$V'' = 10\pi - \frac{10}{3}\pi x \Big| < 0$ ($x = 6$)

$\therefore$ max f'' test

$V = \frac{1}{3}\pi (\overset{12}{36})(15 - \overset{5}{10})$

$V = 60\pi.$

Smaller cone max volume is 60π.

1/4/93

Cmuti 98

~~Dmytriw~~

Dumont 89

Evers 75

Hamilton 65

Keen 98

Mattson 81

Noonan 78

Ryan 78

Advanced Placement Calculus

Name _____

Copy original problem.

Per _____ 93 Date _____

Convince *me* that **you** understand the concept!

Chapter 6 Applications Make–Up Exam

I Differentials. Do not use a calculator for these problems. (15 pts ea.)

a) The expression for the horsepower of an engine is: $P = .4nx^2$, where n is the number of cylinders and x is the bore (diameter) of the cylinders. Determine the approximate increase in power (the differential, itself) when a (4 cylinder) volkswagon motor has the cylinders rebored from 3.250 to 3.265 inches.

b) Calculate $\tan 46^\circ$, approximately, given $1^\circ = .0175$ radians. (hint: $dx \neq 1$. You need to work in radians.)

II At 1 P.M. a bacterial culture weighed 100 grams. At 4:30 P.M. it weighed 250 grams. Assuming that it grows at a rate proportional to the amount present, find at what time (within 5 minutes) it will grow to 400 grams. (20 pts)

III Maximum/minimum problems. Be sure to properly justify.

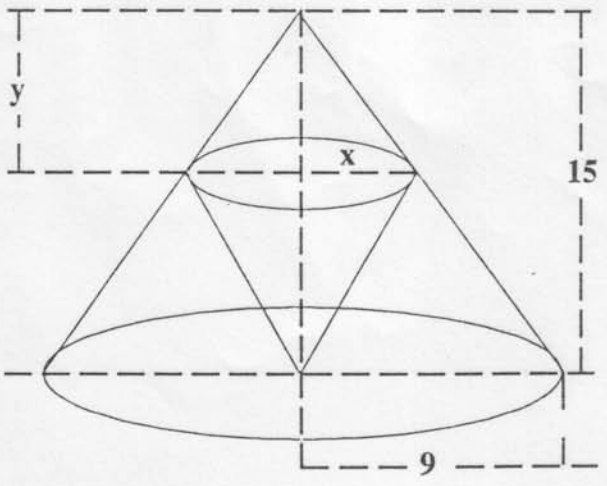
a) A manufacturer of "Sweet Bippies" can sell x items per week at $(200 - .01x)$ **cents each**. The machine that actually produces the Sweet Bippies costs $200.00 (notice dollars) per week for rent and the materials used in the making of **each** Sweet Bippie costs fifty **cents**. (30 pts total)

1) What is tht total production costs (in cents) per week? Write the function which describes *profit* made on the sale of Sweet Bippies.

2) What should be the production level so the manufacturer will have maximum profits? What will be the sales price for each Sweet Bippie at this production level? What will be the profit per week at this level?

3) The government has noticed how well the sales of Sweet Bippies has been. In an effort to reduce "excess profits" the government has imposed an excise tax of ten cents ***per*** Sweet Bippie sold. Since inflation has not affected the manufacturer, all other features of the production, etc. remain unchanged. How much of the ten cents excise tax should be passed on to the consumer to maintain maximum profits for the manufacturer?

b) A right circular cone has altitude 15 feet and a base radius of 9 feet. A smaller cone is inscribed inside the given cone such that its vertex is at the center of the base of the larger cone and its base is parallel to the base of the larger cone. Find the maximum volume of the smaller cone given the volume of a cone is: volume $= \frac{1}{3}\pi r^2 h$. (hint: consider similar triangles to eliminate y.) (20 pts)

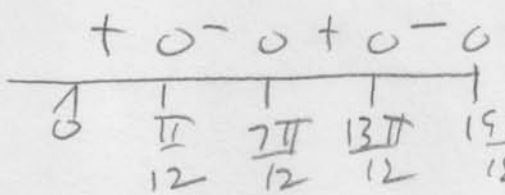


I

$x(\tau) = 5\sin 2\tau + \sqrt{3}\cos 2\tau$

$x' = 2\cos 2\tau - 2\sqrt{3}\sin 2\tau \overset{\text{set}}{=} 0$

$\tan 2\tau = \frac{1}{\sqrt{3}/2}$



$2\tau = \{\frac{\pi}{6}, \frac{2\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}\}$

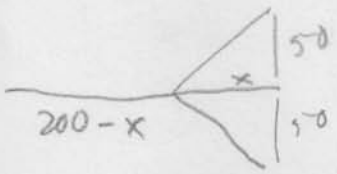
$\tau = \{\frac{\pi}{12}, \frac{2\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}\}$

Period of $x(\tau)$ is π so $x(\frac{\pi}{12}) = x(\frac{13\pi}{12})$ for cos.

$5\sin\frac{\pi}{6} + \sqrt{3}\cos\frac{\pi}{6}$

$\frac{1}{2} + \frac{3}{2} = 2$ = max dist.

II



A) $f(x) = 200 - x + 2\sqrt{x^2+50^2}$

$f'(x) = -1 + \frac{2x}{\sqrt{x^2+50^2}} \overset{\text{set}}{=} 0$

$2x = \sqrt{x^2+50^2}$

$4x^2 = x^2 + 50^2$

$3x^2 = 50^2$

$x = \frac{50}{\sqrt{3}} \approx 29$

$f''(x) = \frac{2\sqrt{x^2+50^2} - \frac{2x^2}{\sqrt{x^2+50^2}}}{x^2+50^2}$

$\frac{2(x^2+50^2) - 2x^2}{(x^2+50^2)^{3/2}}$

$\frac{50^2}{(\quad)^{\frac{3}{2}}} > 0$ min f'' test

Locate the transmitter 171 ft from House P.

B) $C(x) = 2(200-x) + 8\sqrt{x^2+50^2}$

$C' = -2 + \frac{8x}{\sqrt{x^2+50^2}} \overset{\text{set}}{=} 0$

$-2\sqrt{x^2+50^2} = 8x$

$\sqrt{x^2+50^2} = 4x$

$x^2 + 50^2 = 16x^2$

$\frac{50}{\sqrt{15}} = x \approx 13$

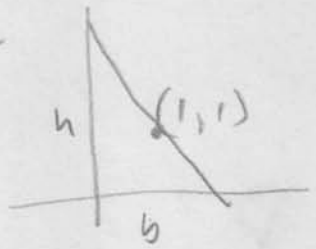
$C'' = \frac{8\sqrt{x^2+50^2} - \frac{8x^2}{\sqrt{x^2+50^2}}}{\sqrt{x^2+50^2}}$

$\frac{8(x^2+50^2) - 8x^2}{(x^2+50)^{3/2}} > 0$

min f'' test.

The transmitter is to be located 187' from House P.

III



$m = -\frac{h}{b}$

$y = -\frac{h}{b}x + h$ | (1,1)

$1 = -\frac{h}{b} + h$

$b = -h + bh$

$\frac{b}{b-1} = h$

$A = \frac{1}{2}bh$

$A(b) = \frac{1}{2} \cdot b \cdot \frac{b}{b-1} = \frac{b^2}{2(b-1)}$

$A'(b) = \frac{4b(b-1) - 2b^2}{4(b-1)^2}$

$= \frac{4b^2 - 4b - 2b^2}{4(b-1)^2}$

$= \frac{2b^2 - 4b}{4(b-1)^2} \overset{SET}{=} 0$

A' : − 0 + (1 | 2)

min f' test

$2b(b-2) = 0$

$b = 0 \quad b = 2$

side lengths

2, 2, $2\sqrt{2}$

IV

$S = -kx^2 \ln x \quad 0 < x < 1$

$S' = (\ln x)(-2kx) - \frac{kx^2}{x}$

$S' = -2kx \ln x - kx \overset{SET}{=} 0$

$-kx(2\ln x + 1) = 0$

$\ln x = -\frac{1}{2}$

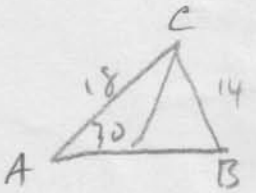
$x = e^{-\frac{1}{2}}$

$S'' = (2\ln x + 1)(-k) - \frac{2kx}{x}$

$-2k$ < 0 max f' test

ratio is $e^{-\frac{1}{2}}$

EXTRA



$\frac{\sin 30}{14} = \frac{\sin B}{18}$

$\frac{1}{28} = \frac{\sin B}{18}$

$\sin B = .6429$

$B = 40°$

$28 = \frac{c}{\sin C}$

A 30°
B 40°
B' 140°
C 110°
C' 10°
a = 14
b = 18
c = 26
c' = 4.9

12/15/94

100	0
90	8
80	5
70	0
60	1
59	7

99

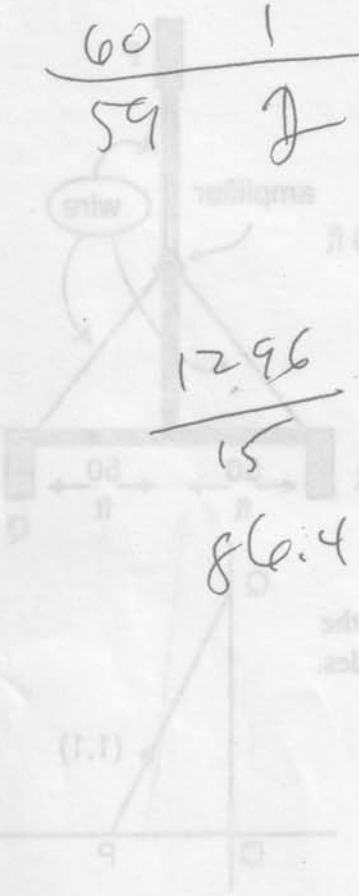
~~56~~

0

1296

15

86.4



Advanced Placement Calculus

Name Calc C6 top 94

Per _____ Date _____

Copy original problem.

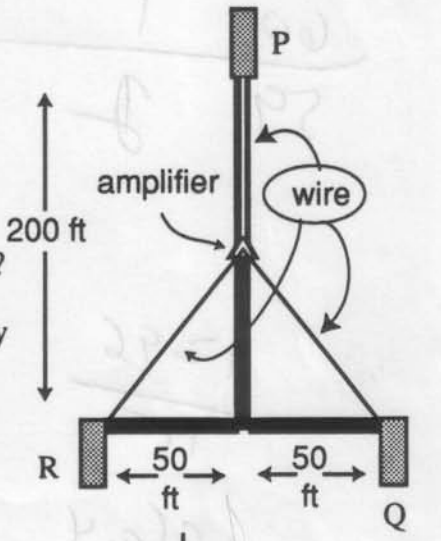
Convince *me* that **you** understand the concept*!*

Each section 25 points.

Chapter 6 Applications Exam

I A mass connected to a spring moves along the x axis so that its x coordinate at time t is given by $x(t) = \sin 2t + \sqrt{3}\cos 2t$. What is the maximum distance of the mass from the origin?

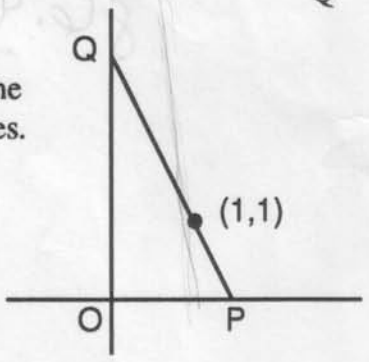
II The cable TV company is planning the location of an amplifier station. They are seeking your advice. The houses are located at P, Q, and R. The amplifier is on the road.



a) Where, to the nearest foot, should the station be located in order to minimize the total length of wire required to service all three houses?

b) While your response to part A is probably correct, the cable company did not act on it before the houses were actually built. Now it costs \$4 per foot to lay cable under private property and \$2 per foot to use the city's conduits running under the street. Where, to the nearest foot, should the amplifier station be located to minimize the cost to the company and still supply service to all three houses?

III Of all triangles that pass through the point (1, 1) and have two sides lying on the coordinate axes, one has the smallest area. Determine the lengths of all its sides.



A) Let OP = b and OQ = h. Write the equation of the line PQ.

B) Substitute (1, 1) into your answer to part A. Solve for h in terms of b.

C) Write a generic function for the area of the triangle OPQ.

D) Write the function for area, $A(b)$ and solve the original question.

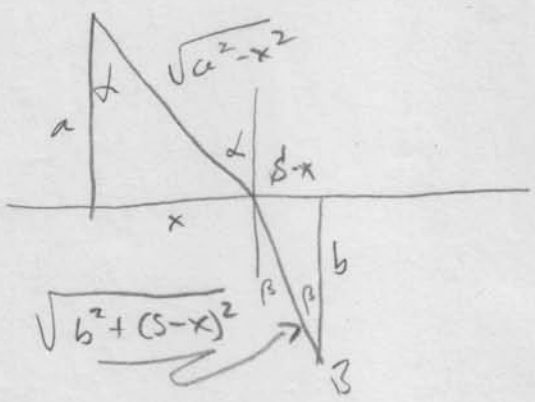
IV In a transatlantic cable, let x be the ratio of the radius of the core to the thickness of the covering. It can be shown that the speed s of a signal passing through the cable is given by:

$s = -kx^2 \ln x$ for $0 < x < 1$ where k is a positive constant. What value of x will maximize s?

Extra Credit ---------- 5 pts ----------

In the triangle ABC, $a = 14$, $b = 18$, and $A = 30^\circ$. Solve for all missing parts.

I



air dist $= \frac{\sqrt{a^2+x^2}}{c}$

air speed

H_2O dist $= \frac{\sqrt{b^2+(S-x)^2}}{v}$

H_2O speed

a) $T(x) = \frac{\sqrt{a^2+x^2}}{c} + \frac{\sqrt{b^2+(S-x)^2}}{v}$

b) $T'(x) = \frac{x}{c\sqrt{a^2+x^2}} + \frac{(S-x)(-1)}{v\sqrt{b^2+(S-x)^2}} = \frac{1}{c}\frac{x}{\sqrt{a^2+x^2}} - \frac{1}{v}\left[\frac{S-x}{\sqrt{b^2+(S-x)^2}}\right]$

c) $T''(x) = \frac{1}{c}\left[\frac{\sqrt{a^2+x^2} - \frac{x^2}{\sqrt{a^2+x^2}}}{a^2+x^2}\right] + \frac{1}{v}\left[\frac{\sqrt{b^2+(S-x)^2} + \frac{(S-x)(2)(S-x)(-1)}{2\sqrt{b^2+(S-x)^2}}}{b^2+(S-x)^2}\right]$

$= \frac{1}{c}\left[\frac{a^2+x^2-x^2}{(a^2+x^2)^{3/2}}\right] + \frac{1}{v}\left[\frac{b^2+(S-x)^2 - (S-x)^2}{(b^2+(S-x)^2)^{3/2}}\right]$

$= \frac{a^2}{c(a^2+x^2)^{3/2}} + \frac{b^2}{v(b^2+(S-x)^2)^{3/2}}$

d) Since $T'' > 0$ $\forall x$, then ANY critical value of T (zero of T') would a min by f'' test.

e) $T'(x) = \frac{x}{c\sqrt{a^2+x^2}} + \frac{(S-x)(-1)}{v\sqrt{b^2+(S-x)^2}}$

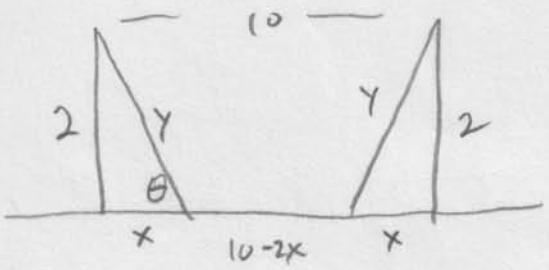
$\sin\alpha = \frac{OPP}{HYP} = \frac{x}{\sqrt{a^2+x^2}}$

$\sin\beta = \frac{OPP}{HYP} = \frac{S-x}{\sqrt{b^2+(S-x)^2}}$

$T'(x) = \frac{\sin\alpha}{c} - \frac{\sin\beta}{v} \overset{\text{Set}}{=} 0$

$\frac{\sin\alpha}{c} = \frac{\sin\beta}{v}$

$\frac{\sin\alpha}{\sin\beta} = \frac{c}{v} = N$



$\frac{x}{2} = \cot\theta \qquad \frac{y}{2} = \csc\theta$

$x = 2\cot\theta \qquad y = 2\csc\theta$

$T = \underset{\text{Road}}{\frac{10-2x}{8}} + \underset{\text{2 forest}}{\frac{2y}{3}}$

$T(\theta) = \frac{10 - 4\cot\theta}{8} + \frac{4\csc\theta}{3}$

$= \frac{5 - 2\cot\theta}{4} + \frac{4\csc\theta}{3}$

$T' = -\frac{1}{2}(-\csc^2\theta) + \frac{4}{3}(-\csc\theta\cot\theta)$

$\frac{\pi}{180}\left[\frac{1}{2\sin^2\theta} + \frac{4\cos\theta}{3\sin^2\theta}\right] \overset{\text{set}}{=} 0$

Since we are using degrees & get this as a decimal we can't set it $= 0$ directly

$3 - 8\cos\theta = 0$

$\cos\theta = \frac{3}{8}$

drop's $\theta = 67^\circ 59'$

$T'(60) = -.0039$ min f'test

$T'(80) = .005$

$T(67^{59}) = 2\text{hr } 29\text{min}$

Time through forest

$\frac{10}{3} = 3\frac{1}{3} = 3\text{hrs } 20$

SAVES 51 minutes.

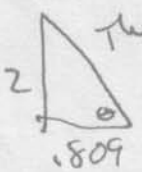
in x

$T(x) = \frac{2\sqrt{x^2+4}}{3} + \frac{5-x}{4}$

$T' = \frac{2x}{3\sqrt{x^2+4}} - \frac{1}{4} \qquad x^2 = \frac{55}{36}$

$x = .809$

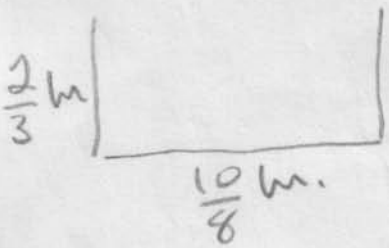
then use Sohcahtoa



Then $\theta = 67.58$

$\tan\theta = \frac{2}{.809}$

EXTRA. Can't use $\theta = 90$ in calculator



$\frac{4}{3} + \frac{5}{4}$

$\frac{16+15}{12} = \frac{31}{12} = 2\text{hrs } 35\text{ min.}$

SAVES only 6 minutes.

This was a great exam.
Generally lossab 12 pts (24%)
on each section if der failed
and generally screwed up.
Some lost as much as 15 — one lost
25 for garbage.
Supisingly good exam!

100	
90	4
80	7
70	7
60	1
59	

99

1551

19

81.6

68

Advanced Placement Calculus Name Calc Ch 6 App

Convince *me* that **you** understand the concept*!*

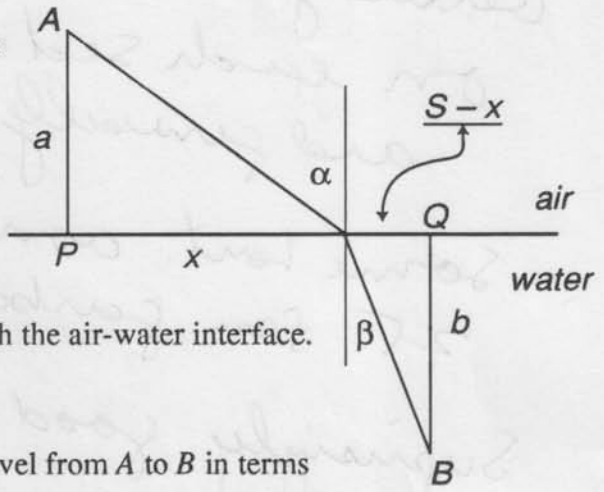
Each section is worth 50 points.

95

Chapter 6 Applications Exam

I

Light travels with speed, c, in a vacuum and almost as fast in air and somewhat slower, v, in water. The constant c is approximately 3×10^{10} cm/sec; the ratio $n = \frac{c}{v}$, known as the index of refraction, depends upon the color of the light, but is approximately 1.33 for water.) In the given figure we show the path of a light ray traveling from point A in air to point B in water, with what appears to be a sudden change in direction as the ray moves through the air-water interface. $\overline{PQ} = S$.



a) Write the time function ,T, required for the ray to travel from A to B in terms of the variable x and the constants a, b, c, S, and v, all of which have been defined or are shown on the figure.

b) Find $T'(x)$.

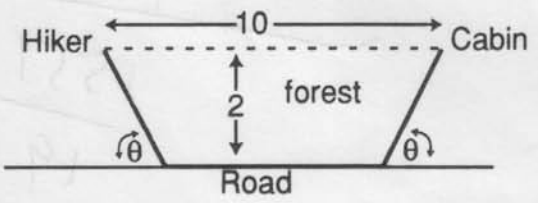
c) Find $T''(x)$.

d) Argue that your answer to part "c" means that *any* zero of T' will be a minimum.

e) Show that $T'(x) = 0$ is equivalent to the condition $\frac{\sin \alpha}{\sin \beta} = \frac{c}{v} = n$. (This is called Snell's Law. The ratio of the sines of the angles of incidence and refraction is equal to the index of refraction.)

II

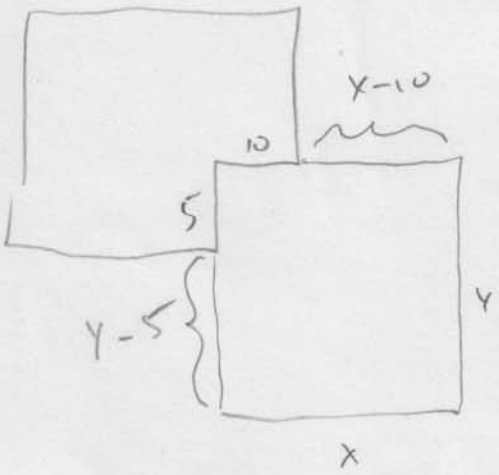
A hiker is in a forest 2 km from a long straight road. The hiker wants to walk to the cabin 10 km away in the forest, and the cabin is also 2 km from the straight road. The hiker can walk 8 km/h along the road, but only 3 km/h through the forest. See the figure. What should be the angle θ shown in the figure in order to minimize the time required for the hiker to get to the cabin? θ is to be in degrees and minutes rounded to the nearest minute. How many minutes will be saved in comparison with the straight route through the forest?



Extra Credit ---------- 5 pts .....

How many minutes are lost in problem II if θ is 90 degrees?

I



$x - 10 + x + y + y - 5 = 85$

$2x + 2y = 100$

$x + y = 50$

$y = 50 - x$

$A = xy$

$A(x) = x(50-x)$

$= 50x - x^2$

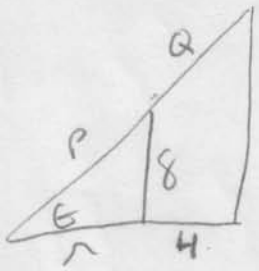
$A' = 50 - 2x \overset{\text{set}}{=} 0$

$x = 25$ $\quad y = 50 - 25 = 25$

$A'' = -2 < 0$ ∴ max f'' test.

Dimensions of pen are 25×25

II



a) WTF $P+Q$

$\sin\theta = \frac{8}{P}$

$P = 8\csc\theta$

$\cos\theta = \frac{A}{P}$

$P\cos\theta = A$

$\frac{A+4}{P+Q} = \cos\theta$

$\frac{A+4}{\cos\theta} = P+Q$

$\frac{P\cos\theta + 4}{\cos\theta} =$

$\frac{8\csc\theta\cos\theta + 4}{\cos\theta} = \frac{8\cot\theta + 4}{\cos\theta} = P+Q$

$Y_1 = (1/\cos x)(4 + 8(\cos x)/\sin x)$

$Y_2 = \text{NDeriv}(Y_1, X, X)$

$Y_3 = \text{NDer}(Y_2, X, X)$

$\text{Solve}(Y_2, X, 50) = 51.5610$

$51^\circ 33$

store $\Rightarrow A$.

$Y_1(A) = 16.65$ Length of Ladder

$Y_3(A) = .0152 > 0$ ∴ min.

OR

$P+Q = \frac{8\cos\theta + 4\sin\theta}{\sin\theta\cos\theta} = f(\theta)$

$u \quad v$

$\frac{\sin\theta\cos\theta(-8\sin\theta + 4\cos\theta) - (8\cos\theta + 4\sin\theta)[\cos^2\theta - \sin^2\theta]}{\sin^2\theta\cos^2\theta}$

$-8\sin^2\theta\cos\theta + 4\sin\theta\cos^2\theta - 8\cos^3\theta + 8\sin^2\theta\cos\theta - 4\sin\theta\cos^2\theta + 4\sin^3\theta \overset{\text{set}}{=} 0$

$8\cos^3\theta = 4\sin^3\theta$

$\tan^3\theta = 2$

$\theta = \tan^{-1}(2\wedge(1/3))$

$51^\circ 33$

$\frac{-8\cos^3\theta + 4\sin^3\theta}{\sin^2\theta\cos^2\theta} \overset{\text{set}}{=} 0$

$4\sin^3 50 - 8\cos^3 50 = -.3265$

$4\sin^3 60 - 8\cos^3 60 = 1.5981$ min first

III $R(x) = kx(M-x)$

$= kMx - kx^2$

$R' = kM - 2kx \overset{\text{Set}}{=} 0$

$x = \frac{M}{2}$

$R'' = -2k < 0 \therefore \max f''(s)$.

Disease spreads fastest when half the population has the disease. Rate of Change.

IV Sales by thousand each.

$P(x) =$ total income less cost for x motors

a) Profit $= 3x - (.04x^2 + .2x + 16)$

$P(x) = -.04x^2 - 2.8x - 16$

$P' = -.08x - 2.8 \overset{\text{Set}}{=} 0$

$x = 35$

$P'' = -.08 < 0 \therefore \max f''(s)$.

Max profit when 35 motors are sold each month.

b) Average cost $= \frac{C(x)}{x} = .04x + .2 + \frac{16}{x} = A(x)$

The cost is a minimum when the production level is 20 motors per month

$A'(x) = .04 - \frac{16}{x^2} \overset{\text{Set}}{=} 0$

$.04x^2 = 16$

$x^2 = 400$

$x = 20$

$A''(x) = \frac{32}{x^3} > 0$

$\therefore \min f''(s)$.

WTP $2^{50} + 2^{50} > 2^{100}$

$2^{50}(1+1) >$

$2^{50} \cdot 2 >$

$2^{51} > 2^{100}$

Reject. This not true

100	2
90	7
80	5
70	3
60	3
59	2

105

35

$\frac{1794}{22} = 81.5$

apparently a very easy test. Kind of hard to grade - each version of it has to be read closely.

Oh well - I was on Jury Duty and this weekend out Nice.

Advanced Placement Calculus

Name Calc Ch 6 App

Copy original problem.

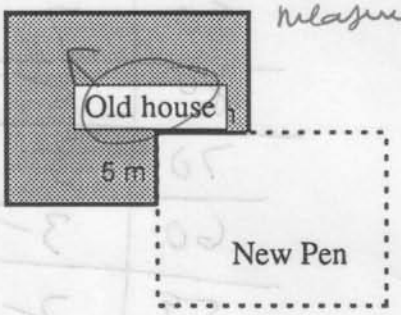
Per 967 Date

Convince *me* that **you** understand the concept!

Each section 25 points.

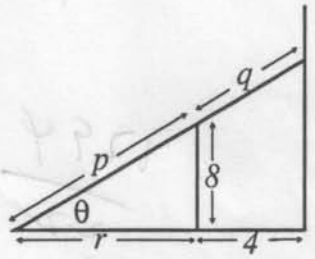
Chapter 6 Applications Exam

I A rectangular outdoor pen is to be added to an existing animal house. See the figure. If 85 meters of new fence are to be used, what should be the dimensions of the new pen in order to maximize its area. Of course, no fence will be used along the walls common to the old animal house.



II An 8 foot high wall is 4 feet from a building. A ladder is leaning across the wall and touches the building.

a) Show that the length of the ladder as a function of θ would be $f(\theta) = \dfrac{4 + 8\cot\theta}{\cos\theta} = \dfrac{4\sin\theta + 8\cos\theta}{\sin\theta\cos\theta}$



b) Use your grapher to determine the length of the shortest ladder as shown. Give a detailed explanation of your method and verify, of course.

III In one model of the spread of a contagious disease among members of a population of M people, the incidence of the disease, R, measured as the number of new cases per day is given in terms of the number, x, of individuals already infected by: $R(x) = kx(M - x)$ where k is some positive constant. How many individuals in the population are infected when the incidence, R is the highest?

IV Chuck Heston at Chariot Motors builds a gasoline engine that delivers about 80 miles per gallon when used to power the standard 2 wheel chariot. Each month, the company sells all of the engines it produces. Thus, the monthly profit earned from selling the engine may be viewed as a function of the monthly production level. Suppose the selling price of each engine is \$3,000, and the total monthly cost of producing x engines is $C(x) = 0.04x^2 + 0.2x + 16$ *thousand* dollars. In reality, the domain of this function is the set of integers. Nevertheless, assume the domain is the interval $[\,0, \infty)$.

a) Find the monthly production level that maximizes the *profit* earned from selling the engine. Profit is defined to be the difference between income from sales and the costs.

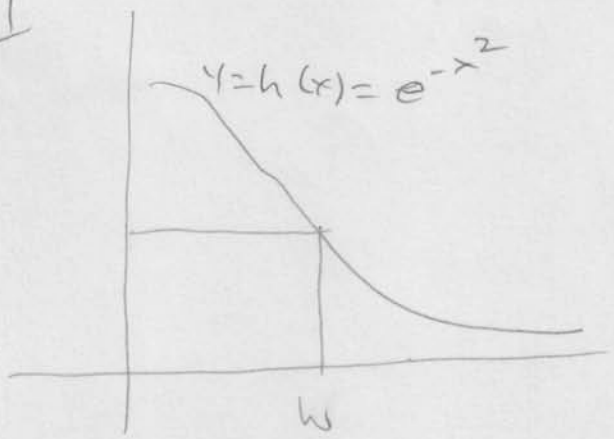
b) Find the monthly production level that minimizes the *average* cost of producing each engine. Average cost is defined to be quotient of the costs divided by the number of sales.

Extra Credit ---------- 5 pts ----------

Prove:

$$2^{80} + 2^{80} > 2^{100}$$

I



$A = l \cdot w$

$A(w) = w(e^{-w^2})$

$A' = e^{-w^2} + w(-2w)e^{-w^2}$

$= e^{-w^2}(1 - 2w^2) \overset{\text{set}}{=} 0$

$w = \frac{1}{\sqrt{2}}$

$+ \quad 0 \quad -$

$0 \qquad \frac{1}{\sqrt{2}}$

max f'.

$h(x) = e^{-x^2}$

$u'(x) = -2x e^{-x^2}$

$h''(x) = e^{-2x}(-2) + 2x(-2x e^{-x^2})$

$= -2e^{-2x} + 4x^2 e^{-x^2}$

$= -2e^{-2x}(1 - 2x^2) \overset{\text{set}}{=} 0$

$x = \frac{1}{\sqrt{2}}$

$h'' \quad + \quad 0 \quad -$

$0 \qquad \frac{1}{\sqrt{2}}$

h'' changes signs at $x = \frac{1}{\sqrt{2}}$ so this is a PI then.

$A(w)$ has a max at $x = \frac{1}{\sqrt{2}}$

so $A(w)$ has its max when h has a PI,

Extra credit

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$x = \frac{\pi}{4}$

$\Delta x = \frac{\pi}{180}$

$\pi \hat{=} 180^\circ$

$\frac{\pi}{180} = 1^\circ$

$46 = 45 + 1$

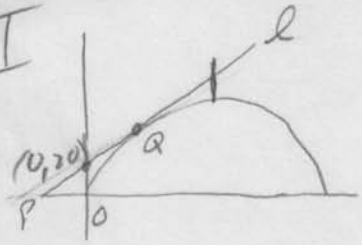
$\frac{\pi}{4} + \frac{\pi}{180}$.

$\tan 46 \approx \tan\frac{\pi}{4} + \left(\sec^2\frac{\pi}{4}\right)\frac{\pi}{180}$

$1 + 2 \cdot \frac{\pi}{180}$

$1 + \frac{\pi}{90}$

II



a) Slope of $\overleftrightarrow{PQ}$ = y' line

$Q(x, x - \frac{x^2}{500})$

$$\frac{x - \frac{x^2}{500} - 20}{x - 0} \overset{\text{set}}{=} 1 - \frac{x}{250}$$

$$\frac{500x - x^2 - 10000}{500x} = \frac{250 - x}{250}$$

$500x - x^2 - 10000 = 500x - 2x^2$

$x^2 = 10000$

$x = 100 \quad x > 0$

The x coordinate of PT Q is 100.

b) Slope of line $y'|_{x=100}$

Slope of line is

$$y'|_{x=100} = 1 - \frac{x}{250}\Big|_{x=100}$$

$$1 - \frac{100}{250}$$

$$1 - \frac{2}{5}$$

$$\frac{3}{5}$$

y intercept is 20 $\therefore$

$l \quad y = \frac{3}{5}x + 20$

c) Tree is at max of y and radius of $y + 50$ ft

$$y = x - \frac{x^2}{500}$$

$$y' = 1 - \frac{x}{250} \overset{\text{set}}{=} 0$$

$x = 250$

max of y when $x = 250$ verified by y'' test.

$$y'' = -\frac{1}{250}$$

$$y|_{x=250} = 250 - \frac{125}{\cancel{250} \cdot \cancel{250}} \cdot \frac{\cancel{500}}{2}$$

$y_{max} = 125$

Tree top = $50 + y_{max}$

$= 175.$

$$y = \frac{3}{5}x + 20\Big|_{x=250}$$

$$\frac{3}{\cancel{5}} \cdot \frac{\cancel{250}}{1} + 20$$

$150 + 20$

$170.$

The light shines on the tree 5 feet from the top of the tree.

100	0
90	2
80	5
70	4
60	2
59	3

94

40

$\frac{1227}{17}$ 72.2

Advanced Placement Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Name Calc C6 AppLic

Per _____ Date 9-7

Chapter 6 Application Exam

I Let $A(w)$ be the area of the shaded rectangle shown in the figure A below. Show that $A(w)$ has its maximum value when w is the x-coordinate of the point of inflection of the graph of h. (50 pts)

figure A

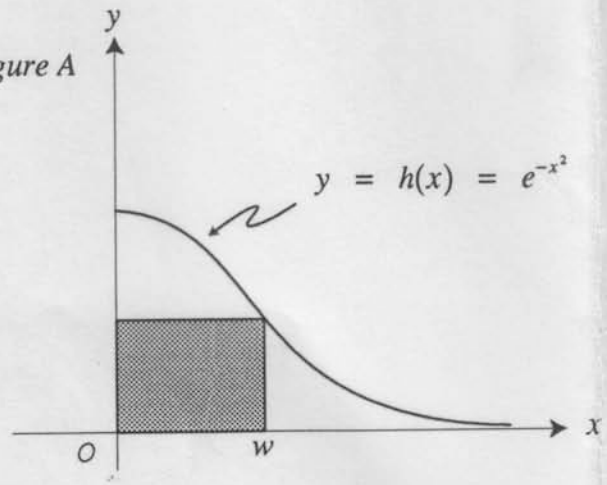
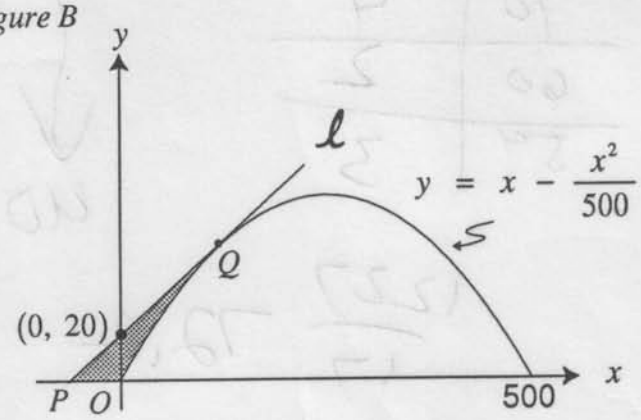


figure B



II Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q, as shown in the figure B above. (50 pts)

a) Find the x-coordinate of point Q.

b) Write an equation for line ℓ.

c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along the line ℓ shine on ~~any part of~~ the tree? Show the work that leads to your conclusion.

Extra Credit ---------- 5 pts ----------

Using differentials, compute the approximation for tan 46°. Do *not* use decimals.

I

a) $\frac{\pi r^2}{2\pi} = \frac{z}{\theta}$

$z = \frac{1}{2}\theta r^2$

b) $r + r + r\theta = 12$

$r(\theta+2) = 12$

$r = \frac{12}{\theta+2}$

$z = \frac{1}{2}\theta\left(\frac{12}{\theta+2}\right)^2$

$= \frac{1}{2}\frac{144\theta}{(\theta+2)^2}$

$z = \frac{72\theta}{(\theta+2)^2}$

c) $z = \frac{72\theta}{(\theta+2)^2}$ u v

$z' = \frac{(\theta+2)^2 72 - 72\theta\, 2(\theta+2)}{(\theta+2)^4}$

$\frac{72(\theta+2) - 144\theta}{(\theta+2)^3}$

$= \frac{72\theta + 144 - 144\theta}{(\theta+2)^3}$

$z' = \frac{144 - 72\theta}{(\theta+2)^3} = 0$ SV

$144 - 72\theta = 0$

$\theta = 2$

	0		2	
	+	0	−	

max at $\theta = 2$

d) $180° = \pi^c$

$\frac{180°}{\pi} = 1^c$

$\frac{360°}{\pi} = 2^c$

$114.6° = 2^c$.

e) $z|_{\theta=2} = \frac{144}{16} = 9$

9 sq feet is max area.

II

a) $P(x) = \begin{cases} A & \text{if } 40 \le x \le 80 \\ B & \text{if } 80 \le x \le C \end{cases}$

$J = 16x$

$(16 - (x-80).08) =$ Profit per block

$16 - .08x + 6.4$

$(22.4 - .08x)x =$ total profit

$K = 22.4x - .08x^2$

m when $22.4x - .08x^2 < 0$ would be upper bound.

$x(22.4 - .08x)$

	0		∞
	+	0	−

$\frac{22.4}{.08} = 280 = m.$

b) $P(x) = \begin{cases} 16x & \text{if } 40 \le x \le 80 \\ 22.4x - .08x^2 & \text{if } 80 < x \le 280 \end{cases}$

only place in question is $x = 80$ since within the subintervals polynomials are continuous.

① $P(80) = 16 \cdot 80 = 1280$

② $\lim_{x \to 80^-} P(x) \stackrel{?}{=} \lim_{x \to 80^+} P(x)$

$1280 = [(22.4 - (.08)80]80$

$1280 = 16 \cdot 80$

③. P is continuous

II $P(x) = \begin{cases} 16x & 40 \le x \le 80 \\ (22.4 - .08x)x & 80 \le x \le 280 \end{cases}$

c) Questionable pt is at $x = 80$

$$\lim_{x \to 80} \frac{f(x) - f(80)}{x - 80}$$

$$\lim_{x \to 80^-} \frac{16x - 1280}{x - 80}$$

$$\lim_{x \to 80^-} \frac{16(x-80)}{x-80} = 16$$

$$\lim_{x \to 80^+} \frac{(22.4 - .08x)x - 1280}{x - 80}$$

$$\lim_{x \to 80^+} \frac{-(.08x^2 - 22.4x + 1280)}{x - 80}$$

$$\lim_{x \to 80^+} \frac{-.08(x^2 - 280x + 16000)}{x - 80}$$

$$\lim_{x \to 80^+} \frac{-.08(x-80)(x-200)}{(x-80)} = 9.6 \neq 16 \therefore$$

limit DNE $\therefore$ Der DNE since limit is DNE by Def of Der.

$x - 200$

$x - 80 \overline{)\, x^2 - 280x + 16000}$

$x^2 - 80x$

$-200x$

$-200x + 1000$

D)

$P(x) = 22.4x - .08x^2$

$P'(x) = 22.4 - .16x \overset{\text{set}}{=} 0$

$x = 140$

$P'(x)$: + 0 −

0, 40, 80, 140, 280

The 4 numbers are 40, 80, 140, 280. Max at 140 by f' test.

$P(140) = (22.4 - .08(140))140$

$(22.4 - 11.2)\,\$$

$11.2(140) = 1568$

max profit to expect is $1,568.

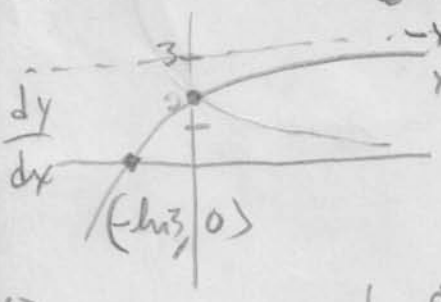
III $y = 3x + e^{-x}$

a) $y' = 3 - e^{-x} \overset{\text{set}}{=} 0$

$e^{-x} = 3$

$-x = \ln 3$

$x = -\ln 3$



b) y increasing when $\frac{dy}{dx} > 0$

$\therefore$ y increases for $x > -\ln 3$

c) $x = 0$

$y|_{x=0} = 0 + 1 = 1$

$(0,1)$ $m = 3 - e^{-0}$

$m = 2$

$y - 1 = 2(x - 0)$

$y = 2x + 1$

d) Normal has slope $-\frac{1}{2}$

$(0,1)$

$y - 1 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x + 1$

97

JD PITNER 70

[illegible] Hildebrand zew

98

[illegible] MARSH assault 84

[illegible] wesp 52

11/5/94

Marie Suga 25

Larry Leung 55

Advanced Placement Calculus

Name 94/97/98

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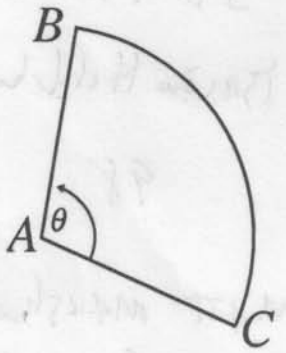
Per _____ Date _____

Convince *me* that **you** understand the concept!

Chapter 6 Make Up Exam

angle MU

I In the diagram, AB and AC are radii of length R for a circle with center at A. The perimeter of the figure is 12 feet. The arc BC of the circle subtends an angle θ radians at A. " θ radians" means that we are describing the angle in radians rather than degrees. A full circle contains a central of 2π radians which we know is the arc length of the unit circle. In this problem the arc length BC is $R\theta$ (50 pts)



a) Write a ratio of the area of a circle with radius r to the angle in radians (2π) of a circle and set it equal to the ratio of the area of the sector (Z) to the central angle θ of the sector. Solve for Z.

b) Using the given information above and part "a", show that the area of the sector of the given figure is: $z = \dfrac{72\theta}{(\theta + 2)^2}$

c) Determine θ so that Z is a maximum.

d) What is the angle θ when measured in degrees (convert the value of θ radians to degrees.). Your answer should be accurate to the nearest minute.

e) What is the maximum area as determined in (a) ?

II In the planning of a coffee shop it is estimated that if there are places for forty to eighty people, the daily profit will be sixteen dollars per place. However, if the seating capacity is above eighty places, the daily profit on each place will decrease by eight cents times the number of places above eighty. Let x places be the seating capacity and $p(x)$ dollars be the daily profit. (The use of decimals in this problem makes the arithmetic easier but you must document your work very carefully. "Answers from nowhere are wrong.") (50 pts)

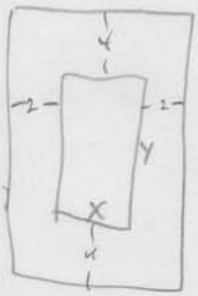
a) Determine $J, K,$ and M and fill in the piece-wise function for $p(x) = \begin{cases} J & \text{if } 40 \le x \le 80 \\ K & \text{if } 80 < x \le M \end{cases}$

b) Even though x, by definition, is a positive integer, to have a continuous function we let x take on all real values in the interval. Prove $p(x)$ is continuous for the entire interval.

c) Prove $p(x)$ is *not* differential at the one questionable spot in the entire interval.

d) There are four values of x which must be considered when determining the maximum of $p(x)$. What are those four numbers? Which yields the absolute maximum? What would the daily profit be?

I



$xy = 50 \qquad y = \frac{50}{x}$

$\text{Area} = (x+4)(y+8)$

$(x+4)\left(\frac{50}{x}+8\right)$

$A(x) = 50 + \frac{200}{x} + 8x + 32$

$A'(x) = -\frac{200}{x^2} + 8 \overset{\text{set}}{=} 0$

$8x^2 = 200$

$x^2 = \frac{100}{4}$

$x = 5 \qquad \therefore y = 10$

$x+4 \times y+10$

Poster was

9×18

$A''(x) = \frac{400}{x^3} > 0 \ \forall x > 0 \ \therefore$ min f'' test.

II $f(x) = x^2 + \frac{c}{x}$

a) $f'(x) = 2x - \frac{c}{x^2} \overset{\text{set}}{=} 0$

$x^3 = \frac{c}{2}$

$x = \sqrt[3]{\frac{c}{2}} \overset{\text{set}}{=} 2$

$\frac{c}{2} = 8$

$c = 16$

$f' \quad - \ 0 \ +$ (at $x = 0, 2$)

$f'(1) = 2 - \frac{16}{1} < 0$

$f'(3) = 6 - \frac{16}{9} > 0$

min f' test.

c) $f'' = 2 + \frac{2c}{x^3} \overset{\text{set}}{=} 0$

$2x^3 = -2c$

$x = \sqrt[3]{-c} \overset{\text{set}}{=} 1$

$c = -1$

$f'' \quad - \ 0 \ +$ (at $x = 0, 1$)

must show change of concavity.

$f''\left(\frac{1}{2}\right) = 2 - \frac{2}{\frac{1}{8}} < 0$

$f''(2) = 2 - \frac{2}{8} > 0$

b) $x = \sqrt[3]{\frac{c}{2}} \overset{\text{set}}{=} -3$

$\frac{c}{2} = -27$

$c = -54$

$f' \quad - \ 0 \ +$ (at $x = -3, 0$)

$f'(-4) = -8 + \frac{54}{-3} < 0$

$f'(-2) = 4 - \frac{54}{-2} > 0$

f test.

D) critical value of f is $\sqrt[3]{\frac{c}{2}}$ if $f''(\cdot) \neq 0$ then we have a max or min

$f''\left(\sqrt[3]{\frac{c}{2}}\right) =$

$2 + \frac{2c}{\frac{c}{2}}$

$2 + 4 = 6 > 0 \ \therefore$ only min

it is not max.

Note: No matter if $c > 0$ or $c < 0$,

$f''(c) > 0 \ \therefore$ min.

III

$V(r) = kr^2(R - r)$

$= kRr^2 - kr^3$

$V' = 2kRr - 3kr^2 \overset{set}{=} 0$

$kr(2R - 3r)$

$2R - 3r \overset{set}{=} 0$

$r = \frac{2R}{3}$

$r \neq 0$ Reject not in range.

$r \in \left[\frac{1}{2}R, R\right]$

$V'' = 2kR - 6kr$

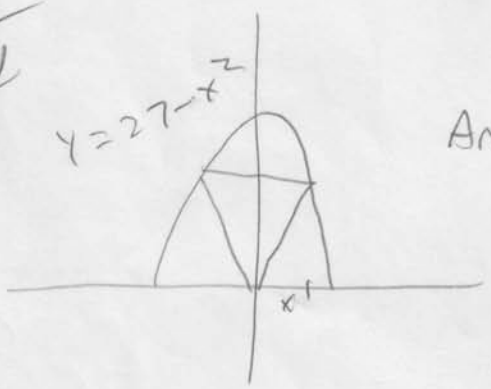
$V''\left(\frac{2R}{3}\right) = 2kR - 6k\left(\frac{2R}{3}\right)$

$2kR - 4kR < 0$

$\therefore$ max f'' test.

The radius of the trachea is $\frac{2}{3}$ rds of the normal Radius during the cough when there is max velocity of air flow.

IV



$Area_\Delta = \frac{1}{2}bh$

$= \frac{1}{2} \cdot 2x \cdot y$

$A(x) = x(27 - x^2)$

$= 27x - x^3$

$A'(x) = 27 - 3x^2 \overset{set}{=} 0$

$x^2 = 9$

$x = \pm 3$

$A''(x) = -6x \Big|_{x=3} < 0 \therefore$ max area f'' test.

$A(3) = 81 - 27$

$= 54$

The largest triangle which can be inscribed the parabola has 54 sq units area.

EXTRA

IT is impossible. I will at 10 mph takes 6 minutes to average 20 mph for 2 miles would required a total of 6 minutes so zero minutes left for 2nd mile

12/11/98

100	0
98	8
80	3
70	2
60	0
59	0

1176

13

90.5

Advanced Placement Calculus Name Calc App Ch6

Copy original problem. Per _____ Date '98

Convince *me* that **you** understand the concept*!*

Chapter 6 Applications Exam

I A poster is to contain fifty square inches of printer matter with margins of four inches at the top and bottom and two inches on each side. Find the overall dimensions of the poster if the total area of the poster is to be a minimum. (20 pts)

II Given $f(x) = x^2 + \frac{c}{x}$ where "c" is a constant. (30 pts tot)

a) Find c such that f has a relative minimum at $x = 2$.

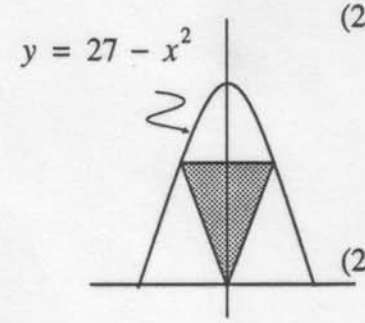
b) Find c such that f has a relative minimum at $x = -3$.

c) Find c such that f has a point of inflection at $x = 1$.

d) Show that f cannot have relative maximum for *any* value of c.

III During a cough there is a decrease in the radius of the person's trachea (windpipe). Suppose that the normal radius of the trachea is R centimeters and the radius of the trachea during a cough is r centimeters, where R is a constant and r is a variable. The velocity of air through the trachea can be shown to be a function of r, and if $V(r)$ centimeters per second is this velocity, then $V(r) = kr^2(R - r)$ where k is a positive constant and r is in $\left[\frac{1}{2}R, R\right]$. Determine the radius of the trachea during a cough for which the velocity of air through the trachea is greatest. (25 pts)

IV Find the area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others above the vertex on a line parallel to the x-axis and on the parabola $y = 27 - x^2$. (25 pts)



Extra Credit ---------- 5 pts .....

On a two-mile race track, you bike the first mile at ten MPH. At what rate must you bike the second mile to average twenty MPH for the entire track?

24.12
288

I Let N = number of weeks held

weight of beef when sold $10{,}000 - 100N = 100(100-N)$

value of beef Now $\frac{16}{10}$ dollars/lb

Price per lb when sold $\left(\frac{16}{10} + \frac{N}{10}\right) = \frac{1}{10}(16+N)$

Storage costs $600N$

$$V(N) = 10\phi(100-N)\cdot\frac{1}{10}(16+N) - 600N$$

$$= 10(1600 + 84N - N^2) - 600N$$

$$= 16000 + 240N - 10N^2$$

$$V' = 240 - 20N \qquad V'' = -20.$$

$V' \overset{\text{set}}{=} 0 \Rightarrow N = 12$ max f'' ⊓, $\therefore$ max f ⊓.

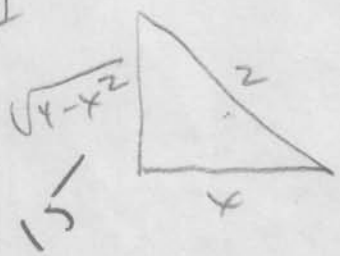
$$16000 - 2880 = 18880 - 1440 = 17440$$

$V(12) = 17440$ (net value when sold.)

$\frac{16}{10} + \frac{12}{10} = \frac{28}{10} = 2.80$ per lb

30

II



15

$A = \frac{1}{2} \overset{u}{x} \overset{v}{\sqrt{4-x^2}}$ want max

$$A' = \frac{1}{2}\sqrt{4-x^2} + \frac{x}{2}\frac{(-x)}{\sqrt{4-x^2}} \overset{\text{set}}{=} 0$$

$4 - x^2 - x^2 = 0$

$x = \pm\sqrt{2}$ reject $-\sqrt{2}$ length > 0.

$A'' = -4x < 0$ $\therefore$ max by f'' ⊓.

$x = \sqrt{2}$

15

III

$Y = x^3 - 3x^2 + 2x - 1$

$Y' = 3x^2 - 6x + 2$

$x = 2$

$dx = .002$

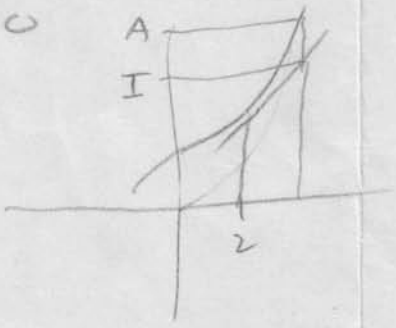
$Y'' = 6x - 6 > 0$

$x = 2$

$$Y \approx -1 + 2(.002)$$

$$\approx -1 - .004$$

$$\approx -1.004$$



	1	−3	2	−1		3	−6	2
2	1	−1	0	−1	2	3	0	2

Since $Y'' > 0$ Y is concave up at $x = 2$ so approx is slightly less than the actual value

IV $f(x) = \cos x$

$f'(x) = -\sin x$

15 $x = \frac{\pi}{3}$

$dx = \frac{\pi}{9}$

$f(x+dx) \approx f(x) + f'(x)\,dx$

$\approx \cos\frac{\pi}{3} - \sin\left(\frac{\pi}{3}\right)\frac{\pi}{9}$

$\approx \frac{1}{2} - \frac{\pi\sqrt{3}}{18}$

$\approx \frac{9 - \pi\sqrt{3}}{18}$

V Problem 13 pg 224

25

$x^2 + h^2 = 4a^2$

$h = \sqrt{4a^2 - x^2}$

$S = k x\left(\sqrt{4a^2 - x^2}\right)^3$

$S' = k(4a^2 - x^2)^{3/2} + k x \frac{3}{2}(-2x)(4a^2 - x^2)^{\frac{1}{2}}$

$= k(4a^2 - x^2)^{\frac{1}{2}}\left[(4a^2 - x^2) - 3x^2\right] = 0$

$4a^2 - 4x^2 = 0$

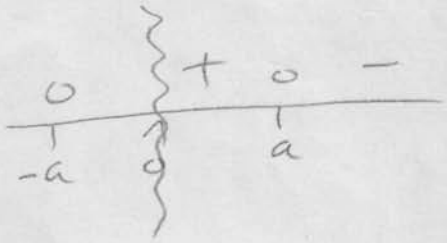
$x = a \quad x > 0.$

$\therefore h = a\sqrt{3}$

using f' crit.

$k(4a^2 - x^2) > 0$ so consider only 2nd factor

if $4(a^2 - x^2) = 4(a-x)(a+x)$



$\therefore$ max. dimensions are

$a \times a\sqrt{3}$

1991

Chris Boston 87
Paul Ventures 82
Michelle Johnson 62
Joann Peters 70
Autenchamber 54

12/15/1999

Budget Lewis 97
Mark Marchwatt 91
Regina Rullo 91
Yohan Hong 85
Dana Nicholas 85
Dylan Ballard 83
Rachel Dutra 77
Tara Holstein 76
Danny Chen 71
Morgan Man 65
TARA Branson 62
Josh Klobas 57

Make warding changes as shown on the 99 version of exam

Advanced Placement Calculus AB

Name Calc 6 Mak.

Per _____ Date _____

Convince *me* that **you** understand the concept!
No Calculators, of course.

99/91/85

Chapter 6 Applications Make Up Exam

I Ten thousand pounds of beef in cold storage are worth one dollar and sixty cents per pound, wholesale. If the price increases steadily at ten cents a pound per week while the beef loses a hundred pounds a week in weight (due to shrinkage) and the storage costs are six hundred dollars per week, how many weeks should the beef be held before selling for the greatest net value? What is that net value? What is the price per pound? when the beef is sold
(tot 30 pts)

II Among all right triangles with hypotenuse of length two, determine (using the methods of calculus) the leg lengths of the right triangle with the largest area. (15 pts)

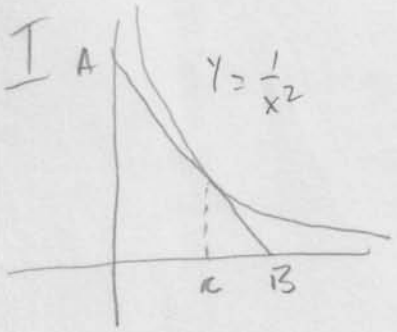
III Find an approximation to the value of ~~y~~ $f(x) = x^3 - 3x^2 + 2x - 1$ when $x = 1.998$ by using differentials. Is your answer greater or less than the actual amount? Explain. (15 pts)

IV Write the linear approximation using the differential of the function $f(x) = \cos x$ near $x = \frac{\pi}{3}$ and use it to approximate the value of $\cos\frac{4\pi}{9}$. (15 pts)

V The stiffness of a rectangular beam is proportional to the product of the width and the cube of the height of its cross section. What shape beam should be cut from a long in the form of a right circular cylinder of radius a in order to maximize its stiffness? (25 pts)

Given $f(x) = x^3 - 3x^2 + 2x - 1$. find an approximation for $f(1.998)$ by using differentials. etc.

I



Need Area. Need x, y intercepts.

$y' = \frac{-2}{x^3}$ $\quad y'(c) = -\frac{2}{c^3}$ $\quad (c, \frac{1}{c^2})$

$y - \frac{1}{c^2} = -\frac{2}{c^3}(x - c)$

$y - \frac{1}{c^2} = -\frac{2}{c^3}x + \frac{2}{c^2}$

$y = -\frac{2}{c^3}x + \frac{3}{c^2} = 0$

$2x = 3c$

$x = \frac{3c}{2}$

$A \to (0, \frac{3}{c^2})$

$B \to (\frac{3c}{2}, 0)$

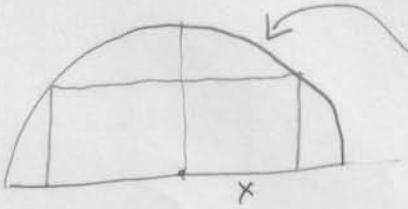
$\text{Area} = \frac{1}{2} \cdot \frac{3}{c^2} \cdot \frac{3c}{2}$

$= \frac{9}{4c}$

$\lim_{c \to \infty} \frac{9}{4c} \to 0$

∴ area is getting smaller.

II



Circle radius 1

$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$

a) $\text{Area} = 2x \cdot \sqrt{1-x^2}$

$A' = 2\sqrt{1-x^2} - \frac{2x \cdot 2x}{2\sqrt{1-x^2}}$

$= \frac{2 - 2x^2 - 2x^2}{\sqrt{1-x^2}}$

$= \frac{2 - 4x^2}{\sqrt{1-x^2}} \overset{\text{set}}{=} 0$

$x^2 = \frac{1}{2}$

$x = \frac{\sqrt{2}}{2}$

Dimension $\frac{\sqrt{2}}{2} \times \sqrt{2}$

	+	0	−
A'			
	0	$\frac{\sqrt{2}}{2}$	

max f'tive.

b) $P = 4x + 2\sqrt{1-x^2}$

$P' = 4 + \frac{2 \cdot (-2x)}{2\sqrt{1-x^2}}$

$= \frac{4\sqrt{1-x^2} - 2x}{\sqrt{1-x^2}} \overset{\text{set}}{=} 0$

$x = 2\sqrt{1-x^2}$

$x^2 = 4 - 4x^2$

$5x^2 =$

$x^2 = \frac{4}{5}$

$x = \frac{2\sqrt{5}}{5}$

	+	0	−
P'			
	0	$\frac{2\sqrt{5}}{5}$	

max f'tive.

Dimensions $\frac{4\sqrt{5}}{5} \times \frac{\sqrt{5}}{5}$

c) It would have been surprising had they been the same, but they weren't and it wasn't.

III $f(x) = (1+\tan x)^{\frac{3}{2}}$

a) Since $1+\tan x > 0$

$1 > -\tan x$

$-1 < \tan x$

$x > -\frac{\pi}{4}$

So Domain is

$-\frac{\pi}{4} \le x < \frac{\pi}{2}$

b) $f'(x) = \frac{3}{2}(1+\tan x)^{\frac{1}{2}} \sec^2 x \ge 0$

$f' > 0$ $\forall$ x in its domain.

So f increases from its endpt.

$f(-\frac{\pi}{4}) = (1-1)^{3/2} = 0$

$(-\frac{\pi}{4}, 0)$ is min. No max since there is no Right endpt.

c) $f'(\frac{\pi}{4}) = \frac{3}{2}(1+1)^{\frac{1}{2}} 2$

$= 3\sqrt{2} = m$

$f(\frac{\pi}{4}) = (1+1)^{\frac{3}{2}} = 2\sqrt{2}$

$y - 2\sqrt{2} = 3\sqrt{2}(x - \frac{\pi}{4})$

$y = 3\sqrt{2}x - \frac{3\pi\sqrt{2}}{4} + 2\sqrt{2}$

d) $f'(0) = \frac{3}{2}(1)(1) = \frac{3}{2}$

$f(0) = 1^{\frac{3}{2}} = 1$

$y - 1 = \frac{3}{2}(x)$

$y = \frac{3}{2}x + 1$

e) $x = 0$, $\Delta x = .02$

$f(x+\Delta x) = f(x) + f'(x)\Delta x$

$= 1 + \frac{3}{2} \cdot \frac{1}{50}$

$= 1 + \frac{3}{100} = 1.03$

IV $f(x) = 2xe^{2x}$

A) $\lim_{x \to -\infty} 2xe^{2x}$

The e^{2x} on the bottom $\to$ large.

limit $\to 0$

$\lim_{x \to +\infty} 2xe^{2x} \to +\infty$

B) $f(x) = 2xe^{2x}$

$f'(x) = 2e^{2x} + 4xe^{2x}$

$= 2e^{2x}(1+2x)$

f': $-$ | 0 | $+$ (at $-\frac{1}{2}$)

min at $x = -\frac{1}{2}$ f'tst.

$f(-\frac{1}{2}) = 2(-\frac{1}{2})e^{2(-\frac{1}{2})}$

$= -\frac{1}{e}$. This is the only min. & absolut.

C) Range is greater than or equal to its abs min

R_f $f(x) \ge -\frac{1}{e}$

Extra. $f(x) = bxe^{bx}$

$f'(x) = be^{bx} + b^2xe^{bx}$

$= be^{bx}(1+bx)$

$f(-\frac{1}{b}) = b(-\frac{1}{b})e^{b(-\frac{1}{b})}$

$= -e^{-1}$

No matter what b is.

f': $-$ | 0 | $+$ (at $-\frac{1}{b}$)

min f'tst.

Argh

Nobody could figure out that they needed x, y intercepts of tangent lines. (I) and They didn't know $x^2+y^2=1$ was the equation of the circle. Argh!

They wanted timid
Chatting. The word
arrogant comes to
mind ...
and the grades drop ....
we lost one Valedictorian
maybe that will be ok!

100	0
90	1
80	2
70	4
60	7
54	9

91

40

$\frac{1461}{23} =$

63.5

Advanced Placement Calculus

Name Cal Ch6 App

Per _____ Date 4/9

Copy original problem.

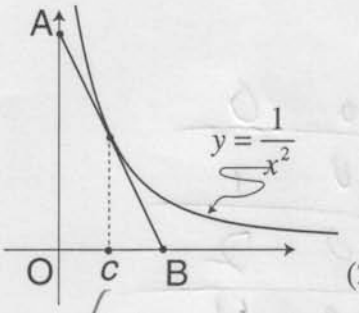
Convince *me* that **you** understand the concept*!*

No Calculators!

Hint: Draw properly labeled, useful figures. Be very clear in your reasoning and answer with a concise, meaningful sentence which indicates that you understand all the concepts involved. Finally, don't waste time justifying mathematics which are incomplete and rambling. Do the job requested and move on.

Chapter 6 Applications Exam

I Does the triangle, AOB, formed by the tangent line to $y = \frac{1}{x^2}$ and the axes increase or decrease in area as c gets larger? Explain. (25 pts)



II Find the dimensions of the rectangle .... (25 pts tot)

a) ...of greatest area that can be inscribed in a semicircle of diameter two.

b) ...of longest perimeter that can be inscribed in a semicircle of diameter two.

c) Is it surprising that the answers to part (a) and part (b) are the same?

(hint a circle of ~~radius~~ diameter 2 centered at the origin is given by $x^2+y^2=1$.)

III Given: $f(x) = (1+\tan x)^{\frac{3}{2}}$. (25 pts tot)

a) Determine k where $k \le x < \frac{\pi}{2}$ is the domain of $f(x)$.

b) Determine the x-value of all extrema.

c) Write the equation of the line tangent to $f(x)$ at $x = \frac{\pi}{4}$

d) Write the equation of the line tangent to $f(x)$ at $x = 0$

e) Using differentials, approximate $f(0.02)$

I a) find the equation of the ~~tangent~~ line AB which is tangent to f(x) at x=c.

b) find the coord of A and of B in terms of c.

IV Let f be the function given by $f(x) = 2xe^{2x}$ (25 pts tot)

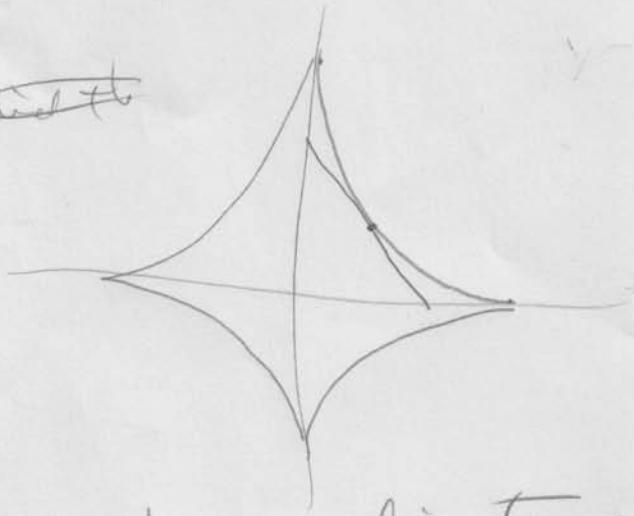
a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.

b) Find the absolute minimum value of f.

c) What is the range of f?

Extra Credit ---------- 5 pts ----------

Continue with problem IV. Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.



$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$

$y^{\frac{2}{3}} = 4 - x^{\frac{2}{3}}$

$y = \sqrt{(4 - x^{\frac{2}{3}})^3}$

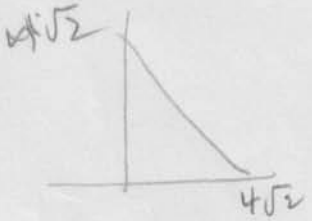
D) Let l be the line tangent to the figure called an "astroid" which is defined by $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$. Find the area of the triangle formed by l and the coordinate axes. Is this a Δ of max area?

$\frac{2}{3} x^{-\frac{1}{3}} + \frac{2}{3} y^{-\frac{1}{3}} \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}} \Big|_{2\sqrt{2}, 2\sqrt{2}} = -1$

$y - 2\sqrt{2} = -1(x - 2\sqrt{2})$

$y = -x + 4\sqrt{2}$



$\frac{1}{2} \cdot 4\sqrt{2} \cdot 4\sqrt{2}$

Area = 16.

given $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$

A a) Solve for y.

B b) Notice that $(-x)$ could be substituted for x and not cause any change in the equation. Furthermore notice that $(-y)$ could be substituted for y and not cause any change. What does this tell us about the graph of the equation?

c) ~~use your graphs to help drawing the graphs of the equations.~~

using a window $x \in [-10, 10]$ and $y \in [-10, 10]$

Draw a sketch of the equations.

D)

$$y = \sqrt{\left(4 - x^{\frac{2}{3}}\right)^3}$$

$x = 1$ $\sqrt{3^{\frac{3}{2}}}$

$3^{\frac{3}{2}}$

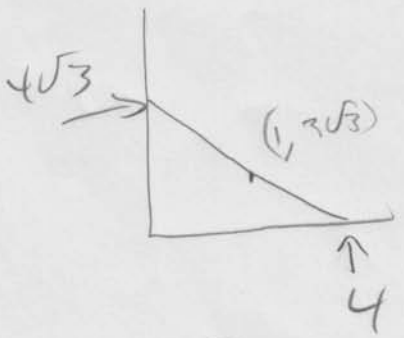
$(1, 3\sqrt{3})$

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

$$= -\left(\frac{3\sqrt{3}}{1}\right)^{\frac{1}{3}}$$

$$= -\left(3^{\frac{3}{2}}\right)^{\frac{1}{3}}$$

$$= -\sqrt{3}$$



$$A = \frac{1}{2} \cdot 4 \cdot 4\sqrt{3}$$

$$8\sqrt{3}.$$

$$y - 3\sqrt{3} = -\sqrt{3}(x - 1)$$

$$= -\sqrt{3}x + \sqrt{3}$$

$$y = -\sqrt{3}x + 4\sqrt{3}$$

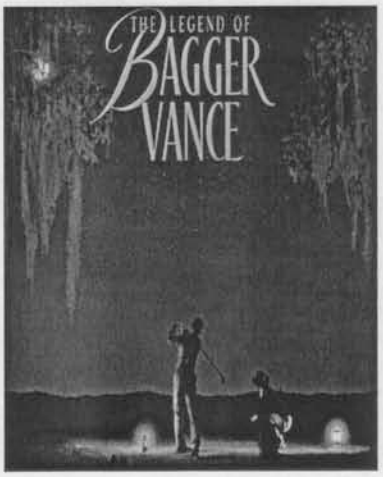
$$0 = \sqrt{3}x = 4\sqrt{3}$$

$$x = 4$$

Calc C 6 app

The Continuing Legend of Bagger Vance

Mr. Mumaugh

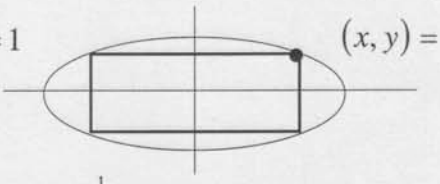


Bagger Vance was first seen in the company of Captain Runnulph Junuh and Harley Greaves. Adele Invergordon provided the setting in Savannah, Georgia in the 1930s and you know the story .....
His legend now continues since Bagger has reappeared in Vacaville. He has seen your calculus work and in his soft spoken style he said "Some things must be learned so they can be remembered." "The rhythm of calculus is just like the rhythm of life." "Find *your* place in the field. Focus. Be there!" "Identify your variable then translate the given into a function." "Draw a useful, labeled picture." "Write down what you *do* know and what you *do not* know. Make the two into one." "Always include justification." "Write a brief explanation of your method." "Convince the reader that *you* know the concept." "It is the closest point because *you* said it was." "When we question the path we've taken, the answer can be right in front of our eyes." "Use a caclulator when necessary. Remember to check your work with a calculator." "3 places after the point accuracy in all decimal answers is expected." "Enjoy."

Bagger sees that you are faced with 18 of the most demanding holes you've ever encountered. He will be there to help but you must remember that "It is your game, not Bagger's," so *you* must rise to the occasion.

1. Given : $f(x) = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$ for $-1 \le x \le 4$. Find all significant points. Sketch graph. Verify your work with your grapher.
2. The volume of 1 kg of water at a temperature T between $0°$ C and $30°$ C is approximately $V = 999.87 - (0.06426)T + (0.0085043)T^2 - (0.0000679)T^3$ cubic centimeters. At what temperature does water have its maximum density?
3. What is the maximum possible area of a rectangle with base that lies on the x-axis and with two upper vertices that lie on the graph of the equation $y = 4 - x^2$?
4. A printing company has eight presses, each of which can print 3600 copies per hour. It costs \$5.00 to set up each press for a run, and $10 + 6n$ dollars to run n presses for 1 hour. How many presses should be used in order to print 50,000 copies of a poster most profitably?
5. A farmer wants to hire workers to pick 900 bushels of beans. Each worker can pick 4 bu/hr and is paid \$1.00/bu. The farmer must also pay a supervisor \$10 per hour while the picking is in progress, and he has miscellaneous additional expenses of \$8 for each worker hired. How many workers should he hire in order to minimize the total cost? What will then be the cost per bushel picked?
6. The heating and cooling costs for a certain uninsulated house are \$500 per year, but with $x \le 10$ in. of insulation the costs would be $\frac{1000}{2+x}$ dollars per year. It will costs \$150 for each inch (thickness) of insulation installed. How many inches of insulation should be installed in order to minimize the *total* (initial plus annual) costs over a 10-year period? What will then be the annual savings resulting from this optimal insulation?
7. A concessionaire had been selling 5000 wieners each game night at 50¢ each. When she raised the price to 70¢ each, sales dropped to 4000 per night. If she has fixed costs of \$1000 per night and each wiener costs her 25¢, what price will maximize her nightly profit?

8. A commuter train carries 640 passengers each day from a suburb to a city. It costs \$1.50 per person to ride the train. It is found that 40 fewer people will ride the train for each 5¢ increase in the fare. What fare should be charged to make the largest possible revenue?
9. Show that the rectangle with maximum perimeter that can be inscribed in a circle is a square.
10. Find the dimensions of the rectangle (with vertical and horizontal sides) of maximum area that can be inscribed in the ellipse with equation $\frac{x^2}{25}+\frac{y^2}{9}=1$



$(x,y)=\left(x,\frac{\sqrt{225-9x^2}}{5}\right)$

11. There is exactly one point on the graph of $y=(3x-4)^{\frac{1}{3}}$ closest to the origin. Find it.
12. A toy manufacturer is planning on mass-producing a new addition to their line of stuffed animals–an eagle. An efficiency expert estimates that if the price per eagle is x dollars, the profit will be given by the formula: $P=K\left(\frac{x-9}{x^2}-0.01\right)$ where $K>0$ for $x>0$. What price maximizes the profit?
13. Find the coordinates of the points on the graph of $f(x)=x^2-1$ which are nearest the origin.
14. Two long, straight railroad tracks cross each other at right angles. Train A, traveling at a constant velocity of 75 mph, crosses the intersection at 5 p.m. southbound on the North-South tracks. Train B, traveling east bound on the East-West tracks at 60 mph, crosses the intersection 10 minutes later. To the nearest second, at what time are the trains closest together?
15. A country whose only export product is coffee can sell x tons of coffee per month on the international market at the price of $\left(300-\frac{x}{1000}\right)$ dollars per ton. The cost of shipping x tons is 10 dollars per ton plus 1000 dollars overhead. Assuming the country has nationalized its coffee industry, what level of export will maximize the dollar income of the country?
16. A rectangle is inscribed in a right triangle, so that two sides of the rectangle lie on the legs of the triangle. Show that the area of the rectangle cannot exceed half the area of the triangle. (hint: *hyp* $y=mx+b$)
17. A farmer has 2040 feet of fencing and wishes to fence off two separate fields. One field is to be a rectangle with the length twice the width, while the other field is to be square. What do you advise the farmer to do so he can enclose the maximum area?
18. A company transports goods from Alamo City to Baker City. Materials are carried in trucks that travel at a speed of x mph. Legal speeds on the road are between 35 and 55 mph. Assume the fuel costs \$1.25 per gal per hour and is consumed at $4+{}^{x^2}\!/_{500}$. The Company pays its drivers \$11 per hour and reimburses them for the cost of fuel as well. You are the efficiency expert. What speed do you recommend the drivers use to minimize the cost per mile?

Bagger knows when it is time to move on. He has seen you prove yourself a worthy competitor and he is very proud of your accomplishments. Alas, even Bagger Vance doesn't know why you get honors credit for a class which is this easy!
Wave to him as he walks into the sunset.
.... knowing full well that you are a better calculus student as a direct result of knowing Bagger Vance and allowing his legend to continue.

Subject: Re: Continuity on open, closed intervals
Date: 11/12 10:08 AM
Received: 11/12 4:18 PM
From: me, me@talmanl1.mscd.edu
Reply-To: Post AP Group, ap-calc@ets.org
To: Post AP Group, ap-calc@ets.org
bcacioppo@hotbot.com

Bob Cacioppo wrote:

> Question: If f is continuous on (a,b) then would it be continuous on
> [a,b]?

Not necessarily. Consider the function

F[x] = 1/((x-a)(x-b)),

which is surely continuous on (a, b), and, just as surely, not continuous on [a,b].

>
> Also, if f is continuous on (a,b), what would have to be true? Would
> we apply the standard criteria of the Definition of continuity to f at
> the points a and b?

Not exactly. See below

> In other words, for continuity on (a,b) to exist:
>
> I. Would f(a) and f(b) have to exist?

Yes.

> II. Would the limit as x approaches a and the limit as x approaches b
> exist?

This is where the convention differs from the standard criterion. We usually accept the right-handed limit at a and the left-handed limit at b. Thus,

G[x] = Sqrt[(a - x) (x - b)]

is generally considered to be continuous on [a, b], although, strictly speaking, the limits at a and at b are not defined. The relevant one-sided limits, however, are both zero.

> III. Would each limit then have to be equal to f(a) and f(b)
> respectively?

We would expect that, at the end-points, the one-sided limits agree with the values of the function. When they do, we say that f is continuous on [a, b].

> I believe that the continuity on (a,b) doesn't guarantee continuity on
> [a,b] since no point beyond a to the left and b to the right is necessary
> to exist for f to be continuous on (a,b). Without a left hand limit for
> a and a right hand limit for b, continuity fails for [a,b] at part II.

If there is a right-hand limit at a, (call it R), we can repair the lack of a left-hand limit at a. We simply define f[x] = R for all x <= a. Similarly, if there is a left-hand limit at b, (call it L), we define f[x] = L for all x >= b. The resulting function is then continuous at both a and b.

--Lou Talman
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http://clem.mscd.edu/~talmanl

Subject: RE: Thank you for replying...
Date: 11/15 8:39 PM
Received: 11/15 8:49 PM
From: Greg Spanier, gspanier@brooks.pvt.k12.ma.us
To: Jim, jamesm98@slip.net

Hi Jim,

Exactly..... I simply meant that applying L'Hopital's rule to limits which are not of the form 0/0 or infinity/infinity can yield incorrect results. I mentioned it because some posters were, in my view incorrectly, applying L'Hopital's rule to your problem (the one you originally stated.) As it happens, I think taking logs then applying (wrongly) L'Hopital's rule happens to give the correct answer in this case.

As far as I know, L'Hopital's rule always gives the right answer when it gives an answer (I'm not sure, but I think there might be cases where it doesn't help.)

As for the problem with x approaching pi/2, once again it seems to me that this is not a L'Hopital situation, since.....

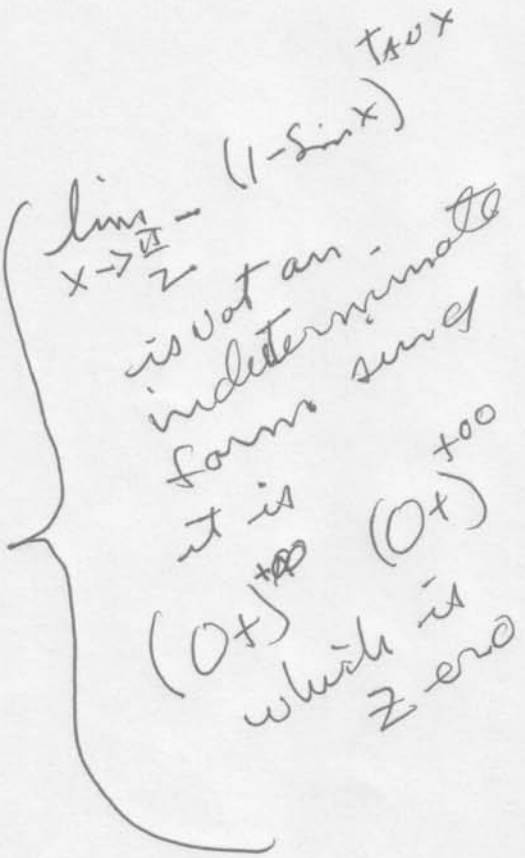
Lim <x -> pi/2 from below> (1 - sin x) ^ tan x is not an indeterminate form, since it is (0+)^(+infinity), which is zero

Note that Lim <x -> pi/2 from above> (1 - sin x) ^ tan x is also not an indeterminate form, being (0+)^(-- infinity), which is + infinity (in fact, the reciprocal of the previous limit, the one from below.) So this limit is undefined by reason of being infinite.

If the problem in fact read Lim <x -> pi/2> (1 - sin x) ^ cot x, this would be of the form 0^0, so taking natural logs, arranging as a quotient and applying L'Hopital's rule will work. I think you have to simplify after one application of L'Hopital's rule and then apply it again. The limit in this case is 1.

Hope this helps,

Greg



-----Original Message-----
From: Jim [mailto:jamesm98@slip.net]
Sent: Monday, November 15, 1999 7:27 PM
To: Greg Spanier
Subject: Thank you for replying...

You said:
<<The function quoted, (1- sin x) ^ tan x is, in fact, continuous at x = 0, so
the limit is simply 1^0 = 1. Note that using techniques like L'Hopital's rule in such situations can yield incorrect results.

And if the exponent is, in fact, cot x, the limit is indeed 1/e.

Greg>>

the original limit as stated was: lim x -> 0 from below of (1 - sin x) ^ tan x.

I wanted to be sure that I understood your comment. I think that the reason L'Hopital's rule might yield incorrect results is because the limit as I had originally stated it was "not a L'Hopital" situation (as I call it). That is the limit was not a quotient of 0/0 or infinity/infinity.

Is this correct or are there L'Hopital situations which still lead to improper results?

With the change from x -> 0 to x -> pi/2 the limit becomes 0^0 which I think usually can be "man-handled" into being a L'Hopital situation. Would the limit be undefined now? ie lim x -> pi/2 from below of (1 - sin x) ^ tan x.

Thank you for helping.

Jim Mumaugh
Vacaville High School
jamesm98@slip.net

Subject: Re: Limits of Compositions
Date: 11/10 3:58 PM
Received: 11/12 4:18 PM
From: Alan Lipp, alipp@crocker.com
Reply-To: Post AP Group, ap-calc@ets.org
To: Post AP Group, ap-calc@ets.org

>Michael Barrientes wrote:
>
>> We just started a 27 week Cal AP course a couple of weeks back and I
>> ran across some problems that made my kids and me think for a bit.
>> They are not satisfied until they know the answer for sure. Here it
>> is:
>>
>> Let f(x) = { 0 if x is not 0
>> pi if x is 0
>>
>> Find the lim x->0 f(sin (x)) and
>> lim x->0 f(f(x)).
>>

Simply compute the composed functions.

f (sin x) = 0 when x is not an integral multiple of pi
= pi when x is an integral multiple of pi

So f(sin x) is the constant function 0 with holes at integral multiples of pi. In particular, f(sin x) has a hole at (0, 0) and so lim f(sin x) = 0 as x__> 0

f (f (x)) = pi when x is not 0
= 0 when x = 0

So f (f (x)) is the constant function pi with a hole at (0, pi). Therefore, lim f (f (x)) = pi as x --> 0.

Alan Lipp

I don't know it ? This is correct.